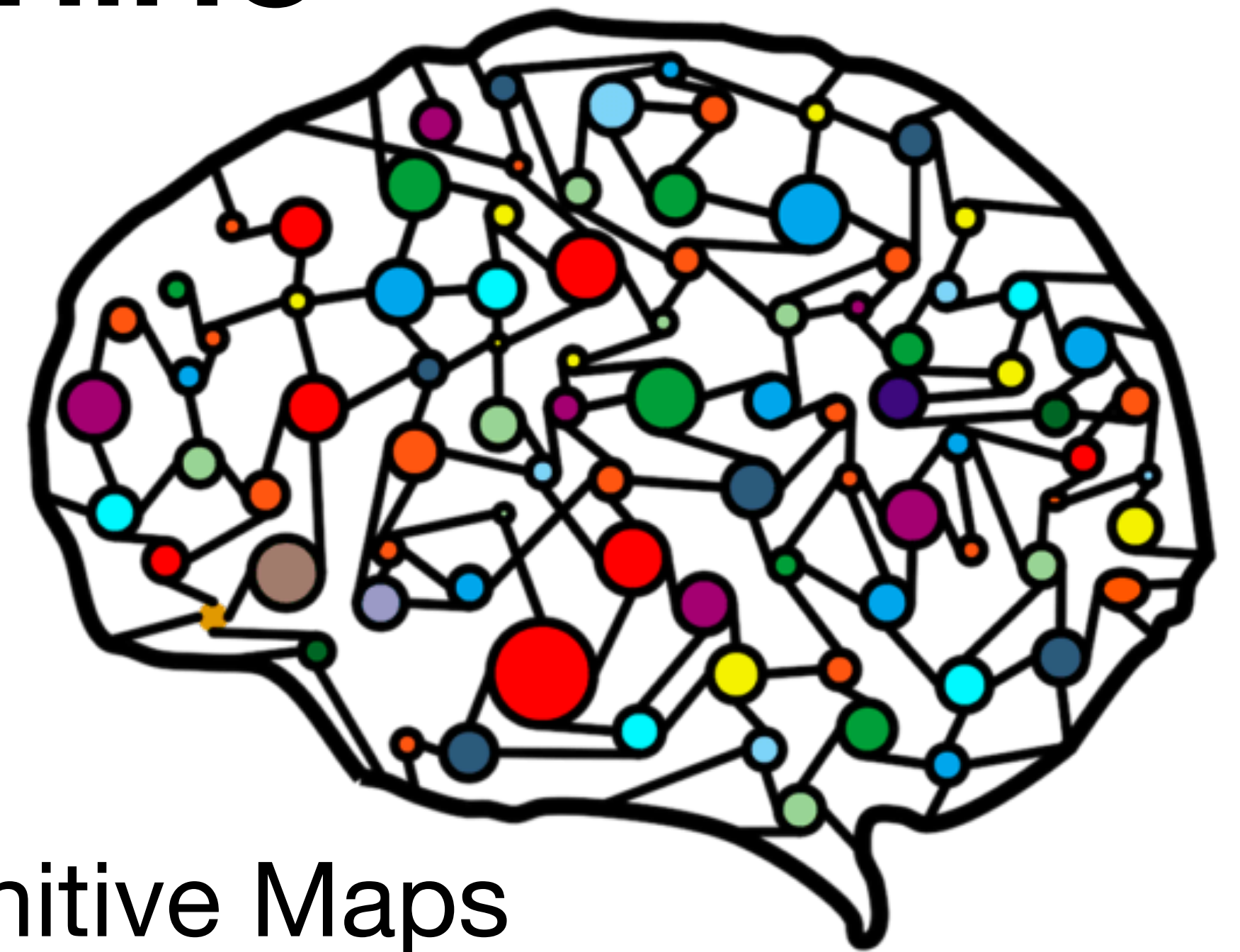


# General Principles of Human and Machine Learning



Lecture 3: Symbolic AI and Cognitive Maps

Dr. Charley Wu

<https://hmc-lab.com/GPHML.html>

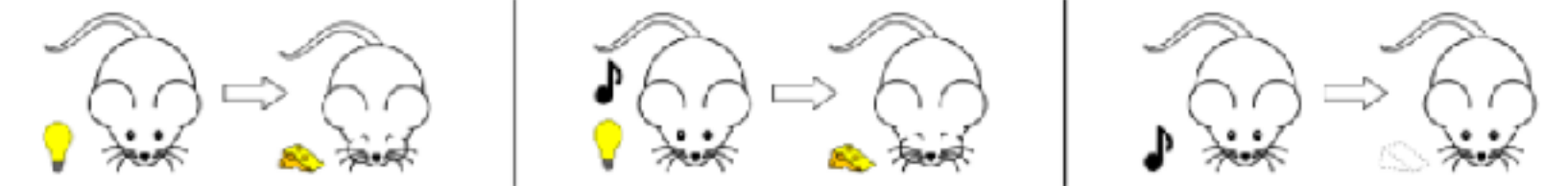
# Clarification from last week's tutorial

- Rescorla Wagner updates: Weights are only updated when the stimuli is present

For  $i$  where  $CS_i = 1$ :

$$w_i \leftarrow w_i + \eta(r_t - \hat{r}_t)$$

Blocking



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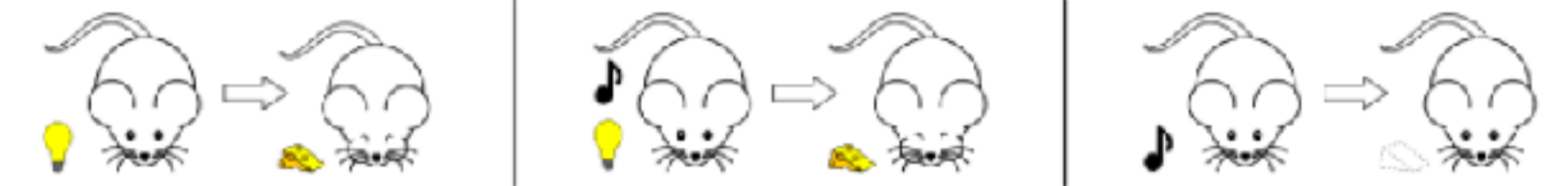
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Learning rate      Observed outcome      Predicted outcome

Blocking

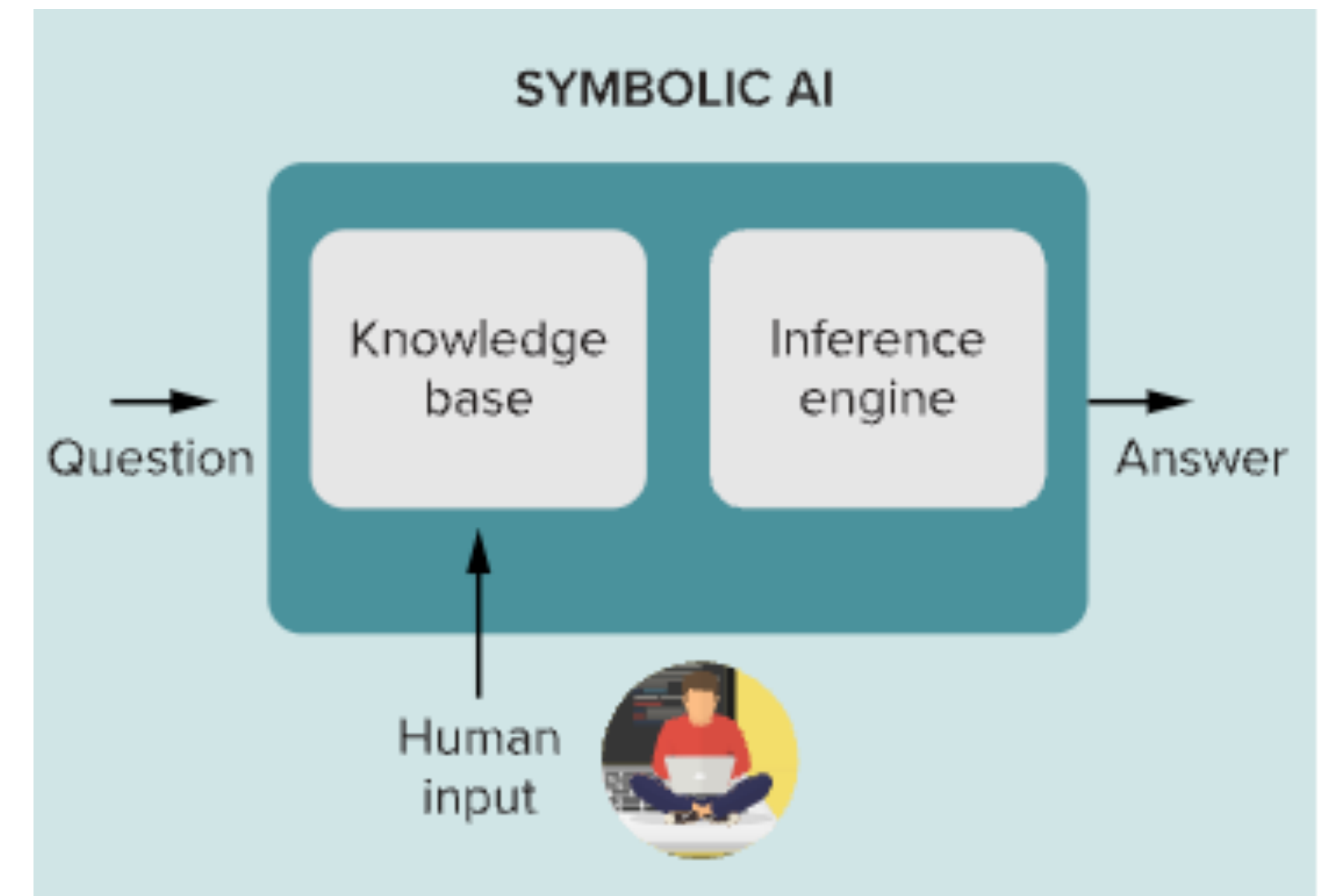




# Lecture Plan

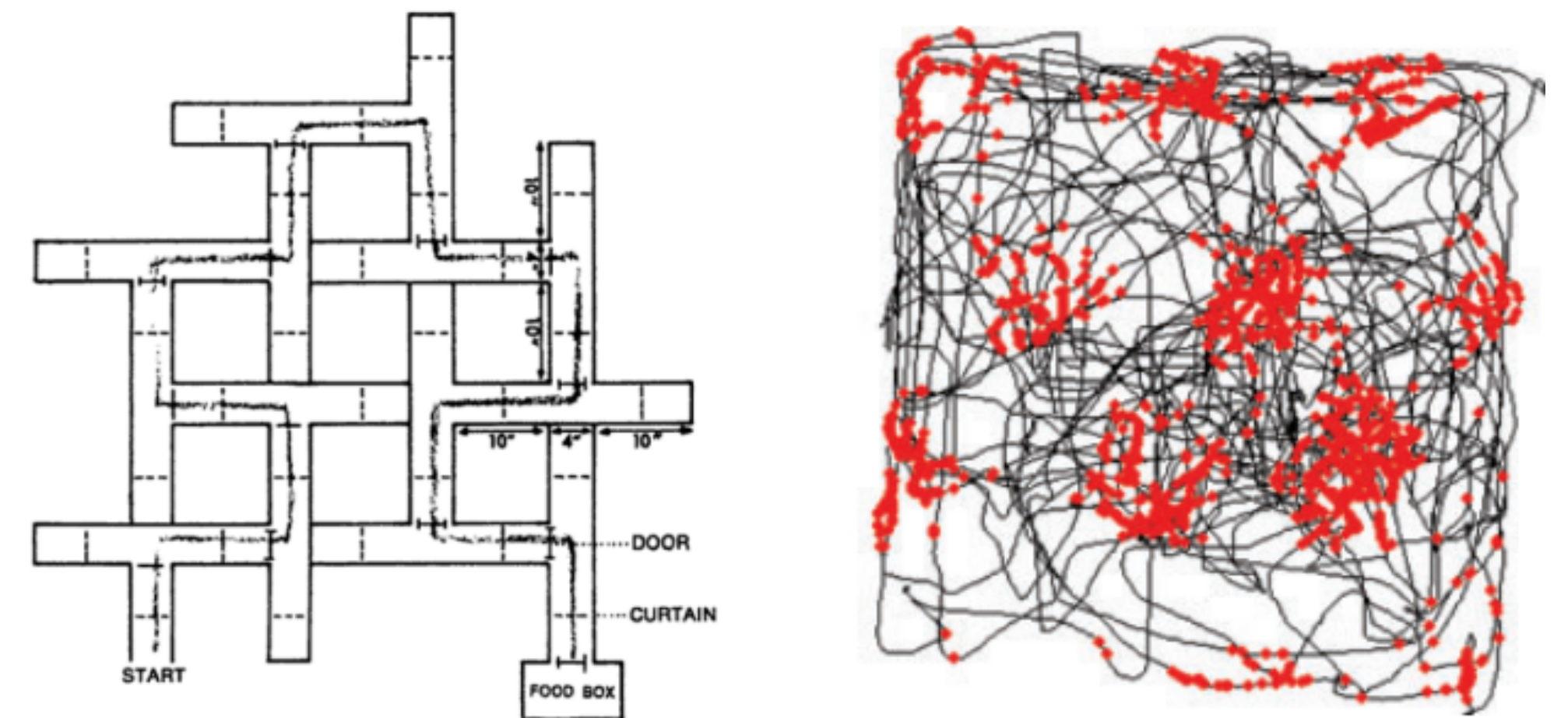
## Symbolic AI

- What happened during the AI winter?
- Intelligence as manipulating symbols through rules and logical operations
- Learning as search



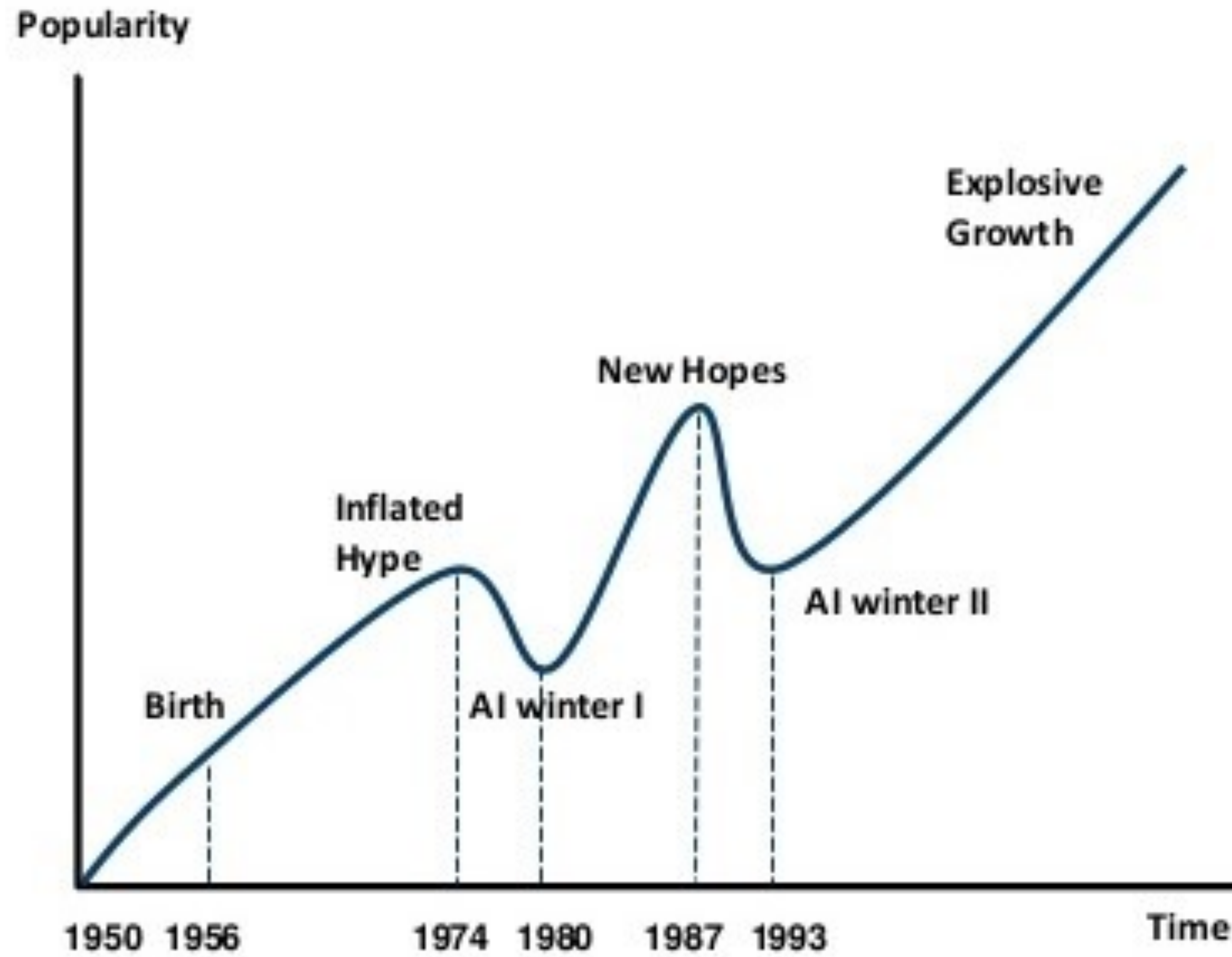
## Cognitive Maps

- From Stimulus-Response learning to Stimulus-Stimulus learning
- Constructing a mental representation of the environment
- Neurological evidence for cognitive maps in the brain

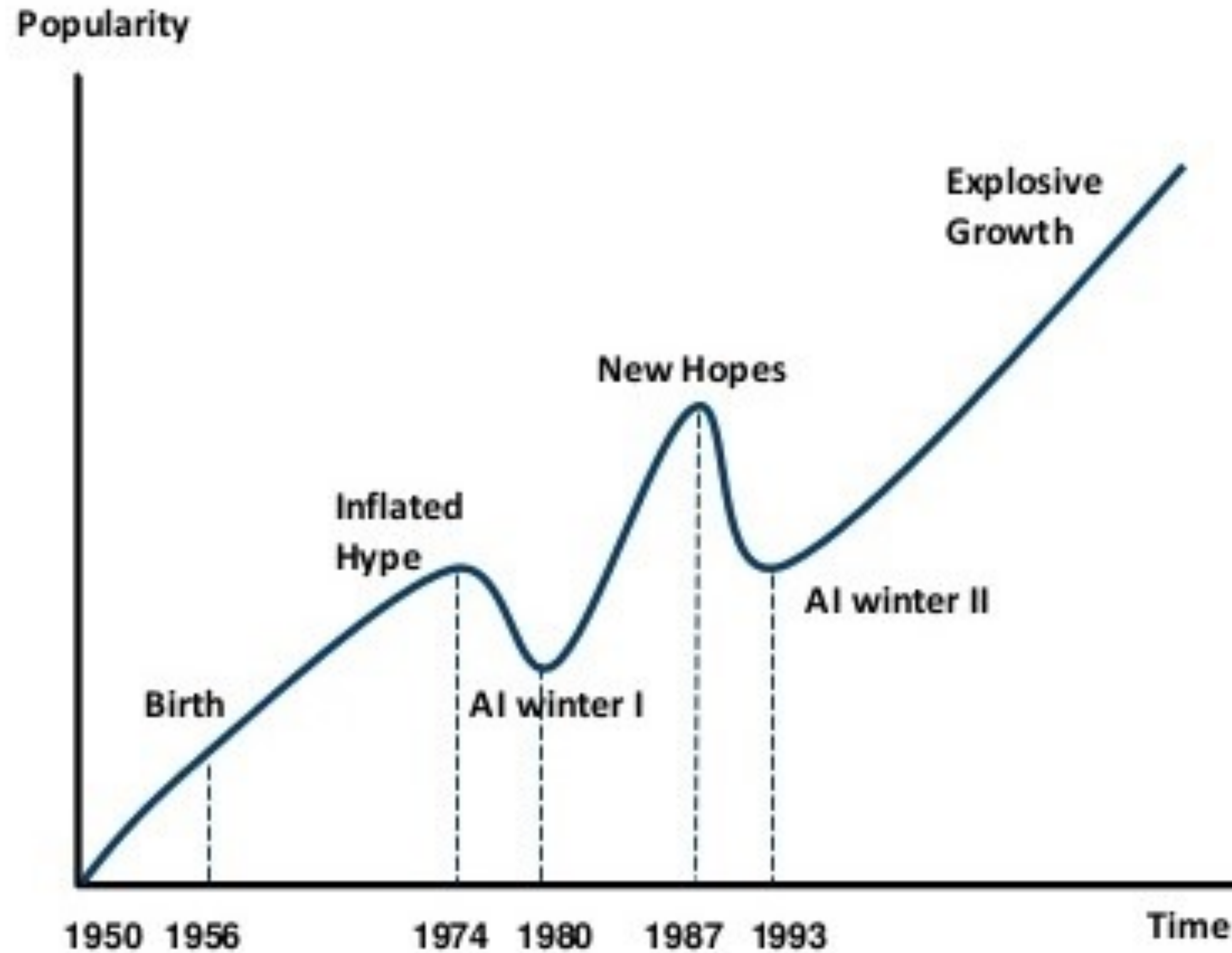




# Timeline of AI

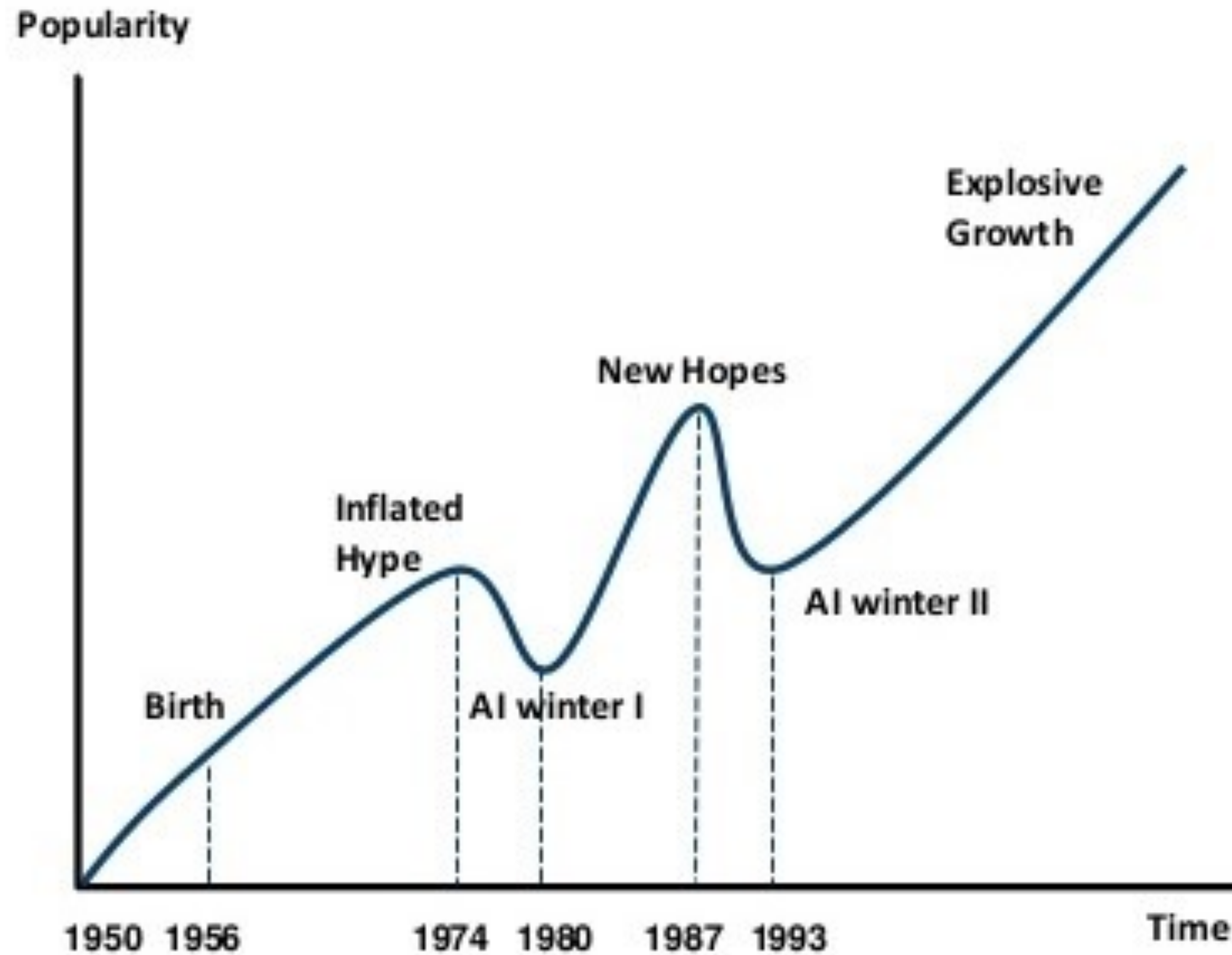


# Timeline of AI



- AI has a long history of being “the next big thing”

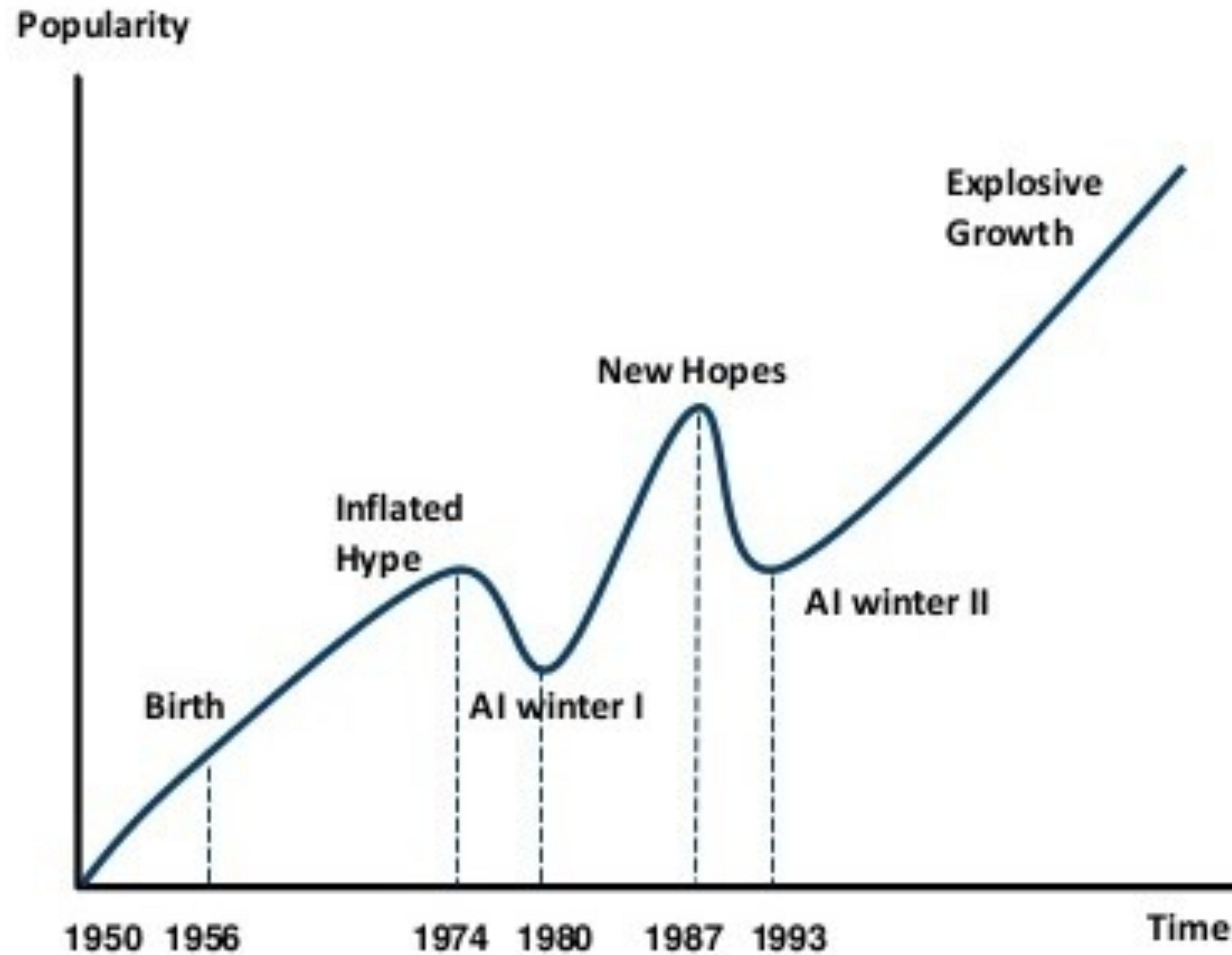
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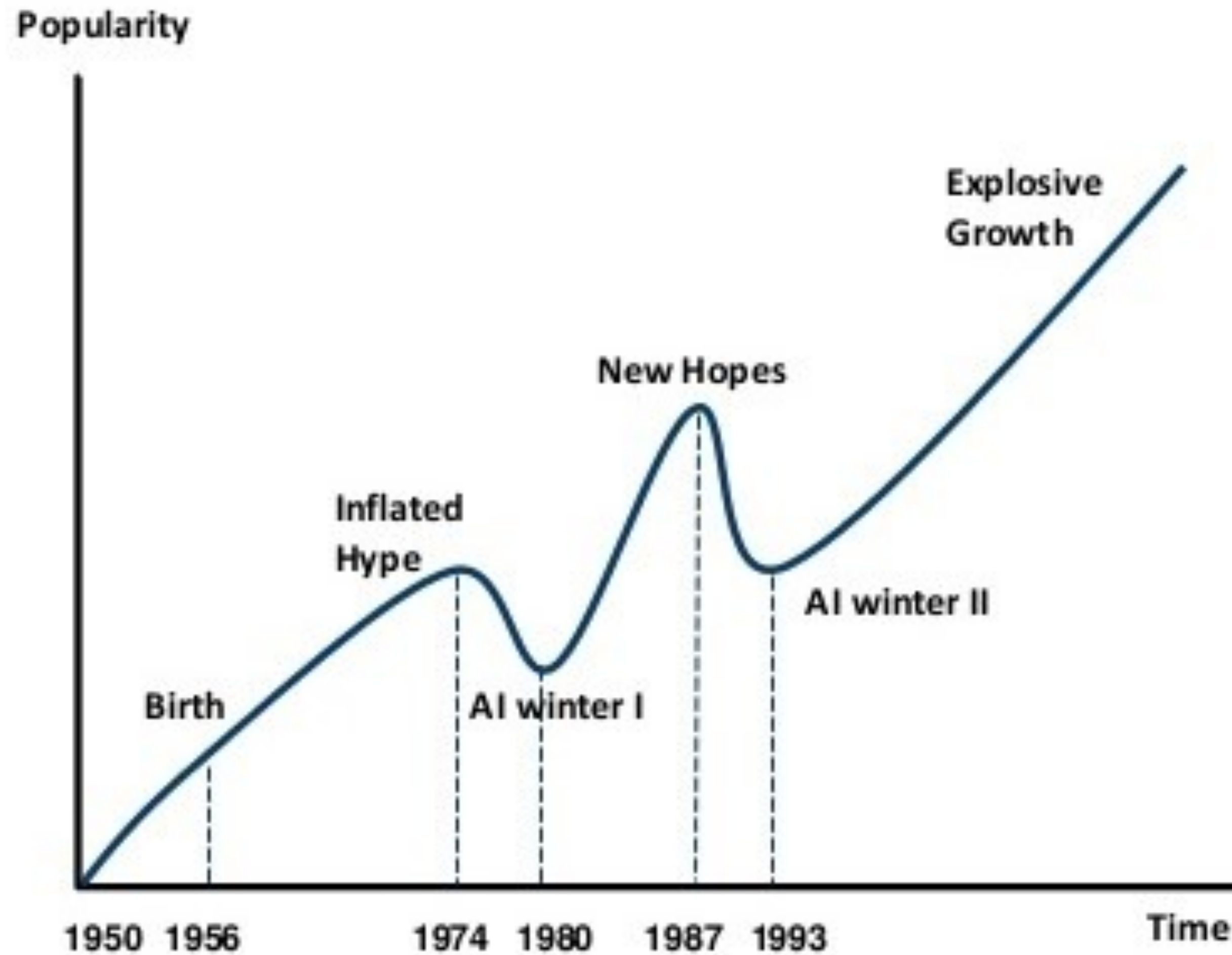


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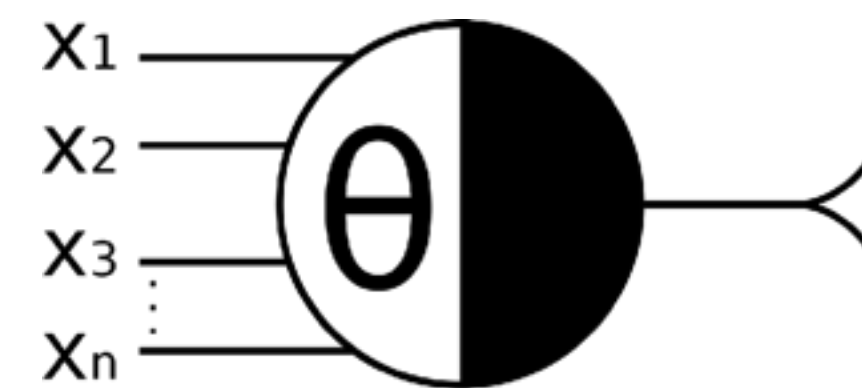


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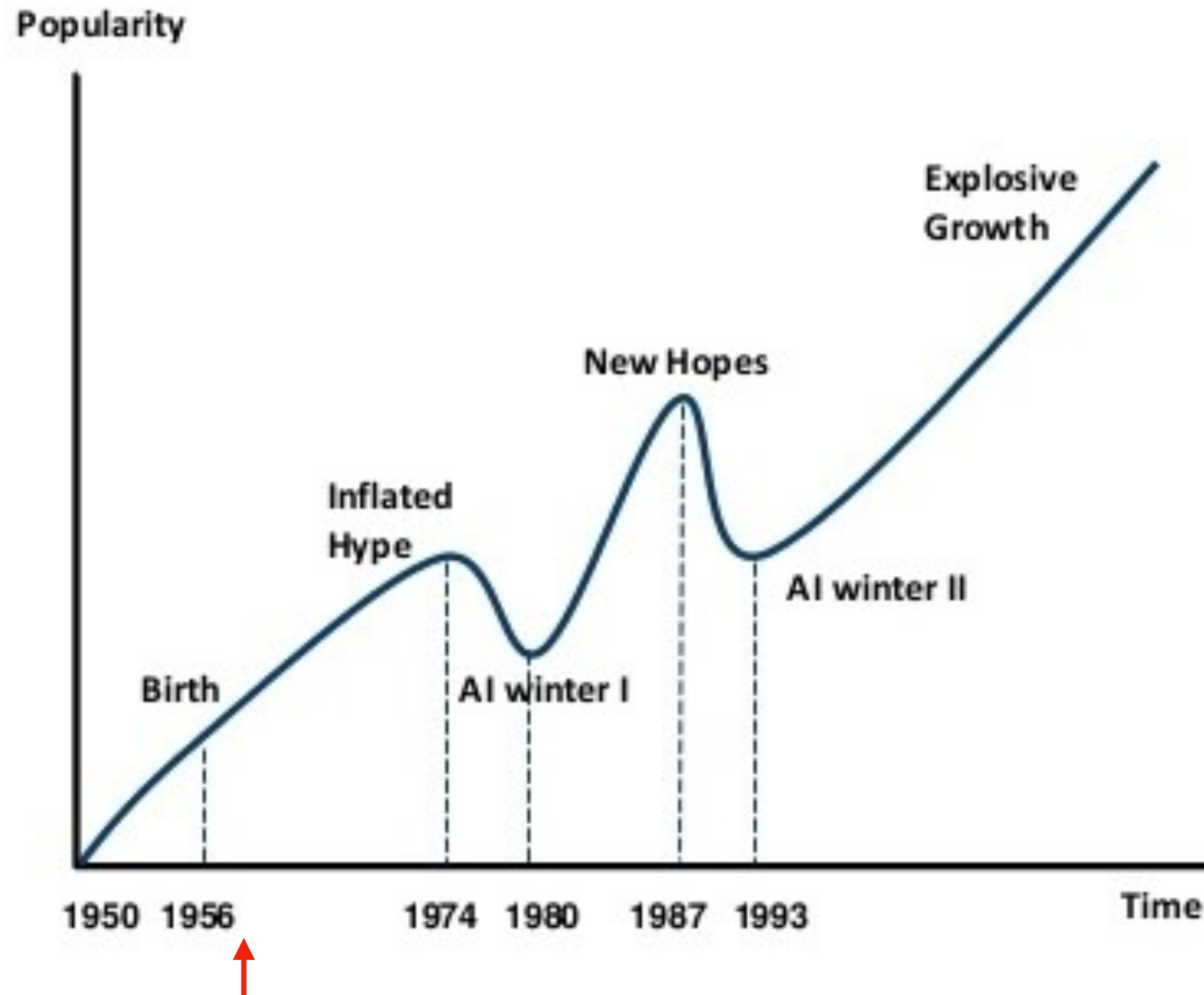


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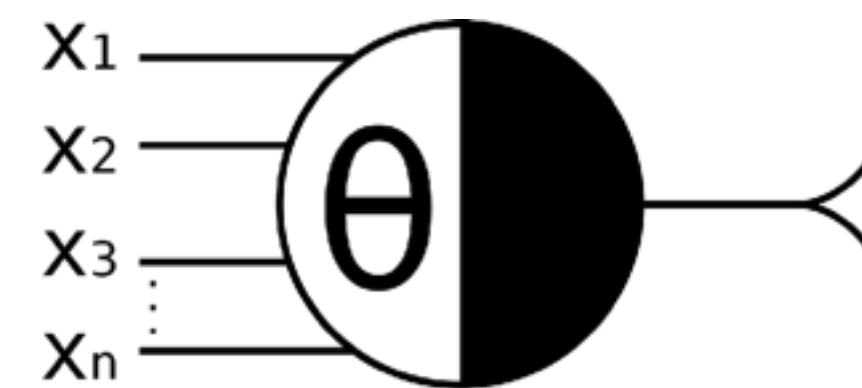


McCulloch & Pitts (1943)  
Perceptron

# Timeline of AI



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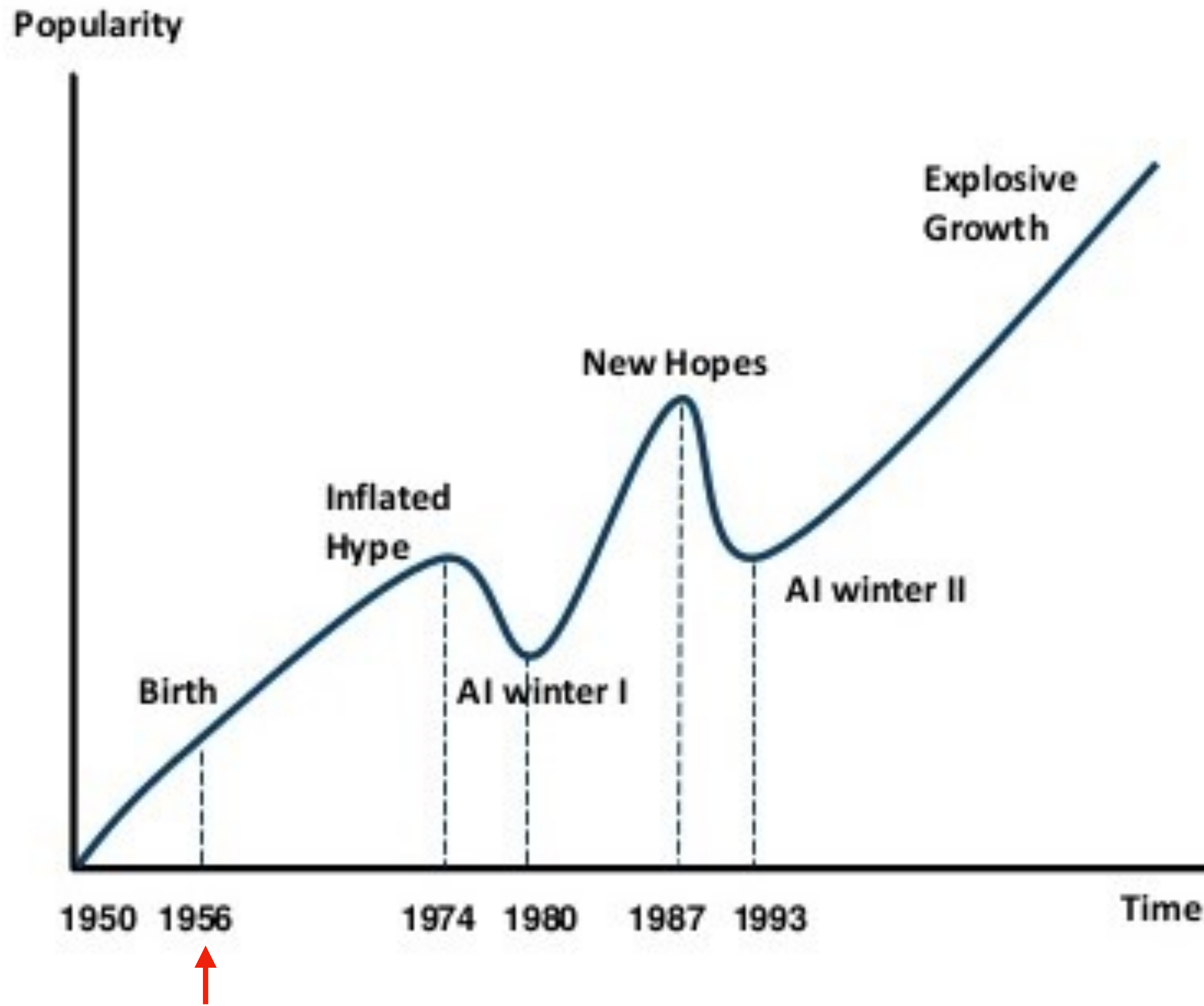
McCulloch & Pitts (1943)  
Perceptron



Rosenblatt (1958) Perceptron



# Timeline of AI

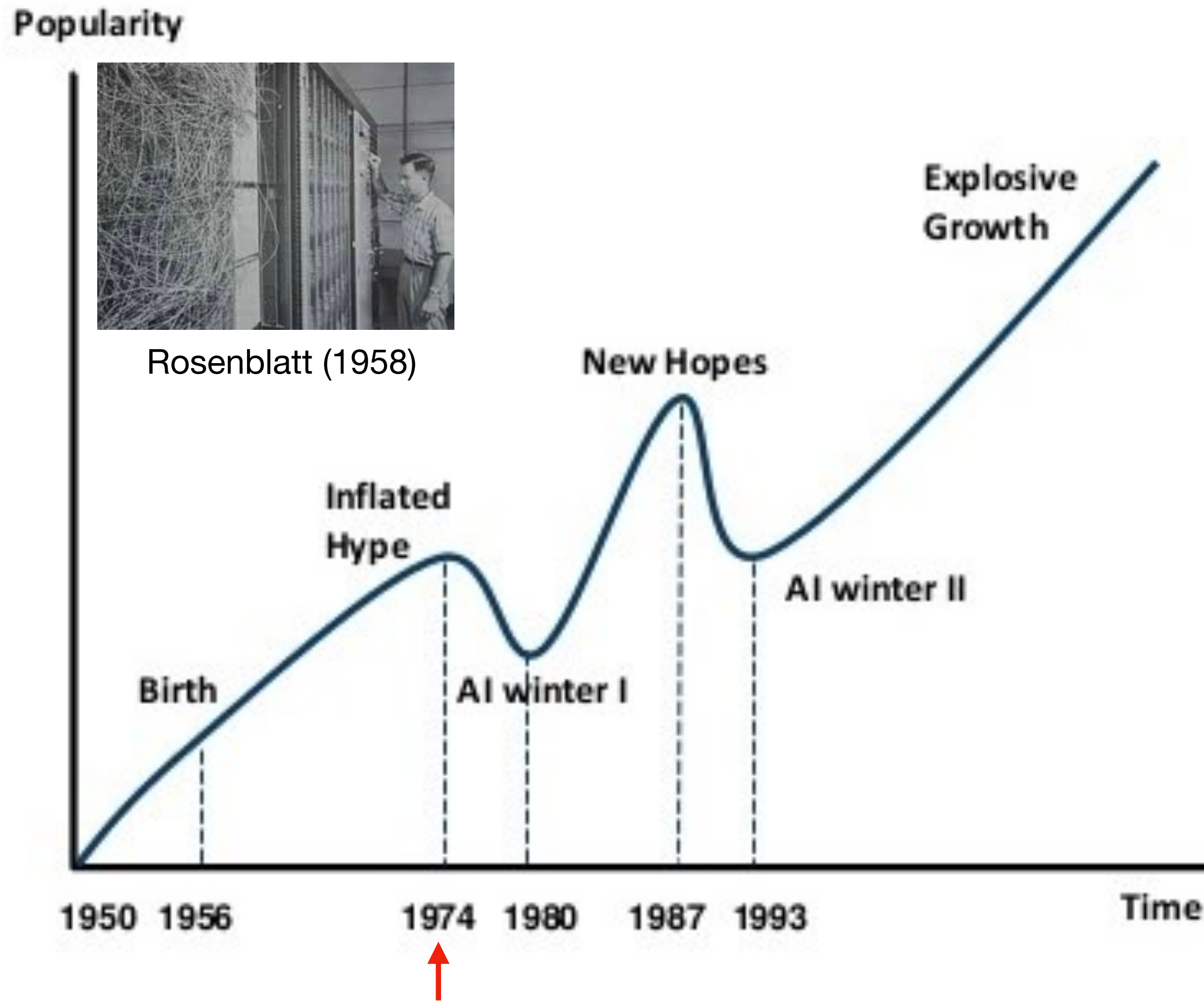


- **1956** Dartmouth workshop considered to be the founding event for AI as a field





# Timeline of AI

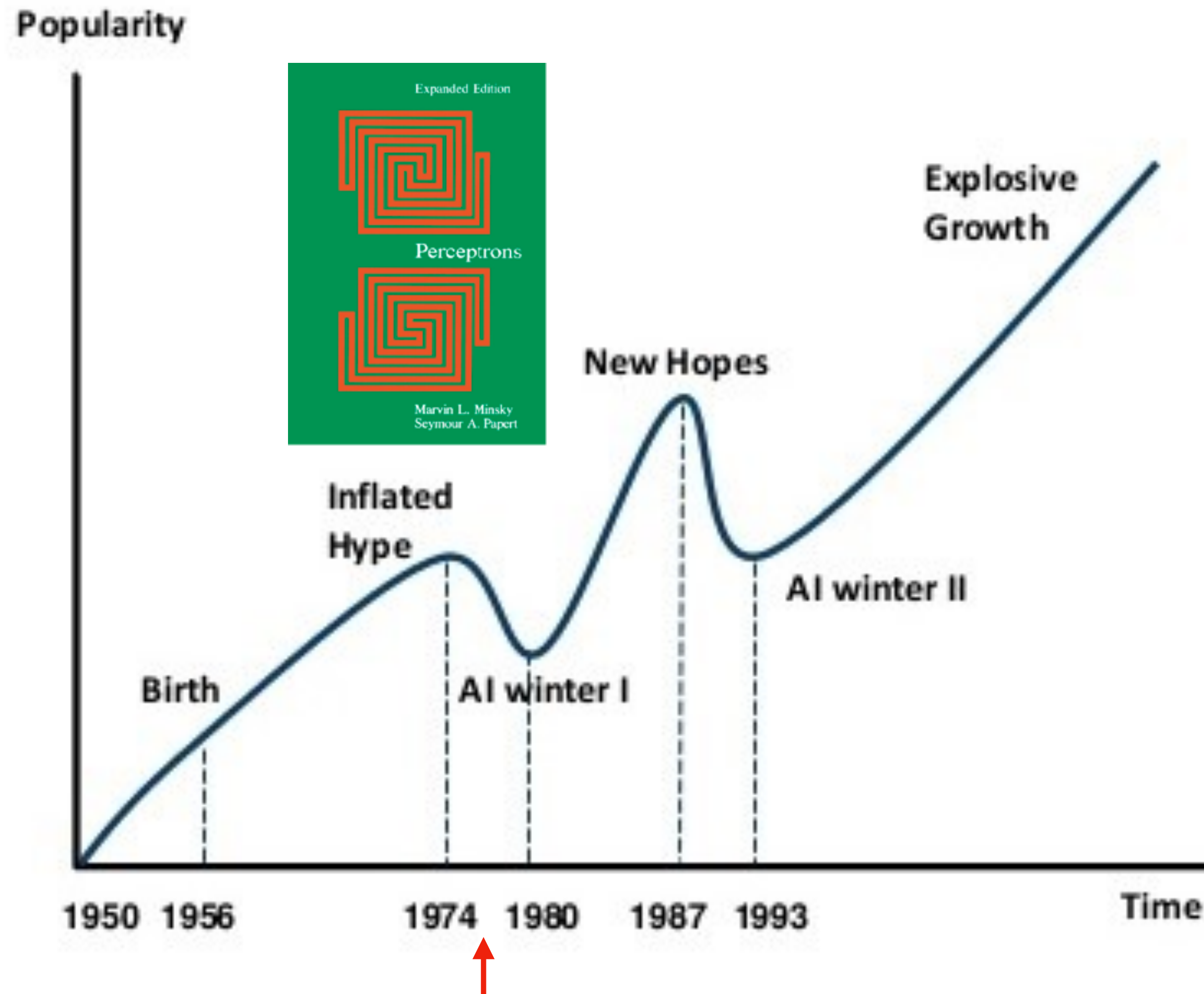


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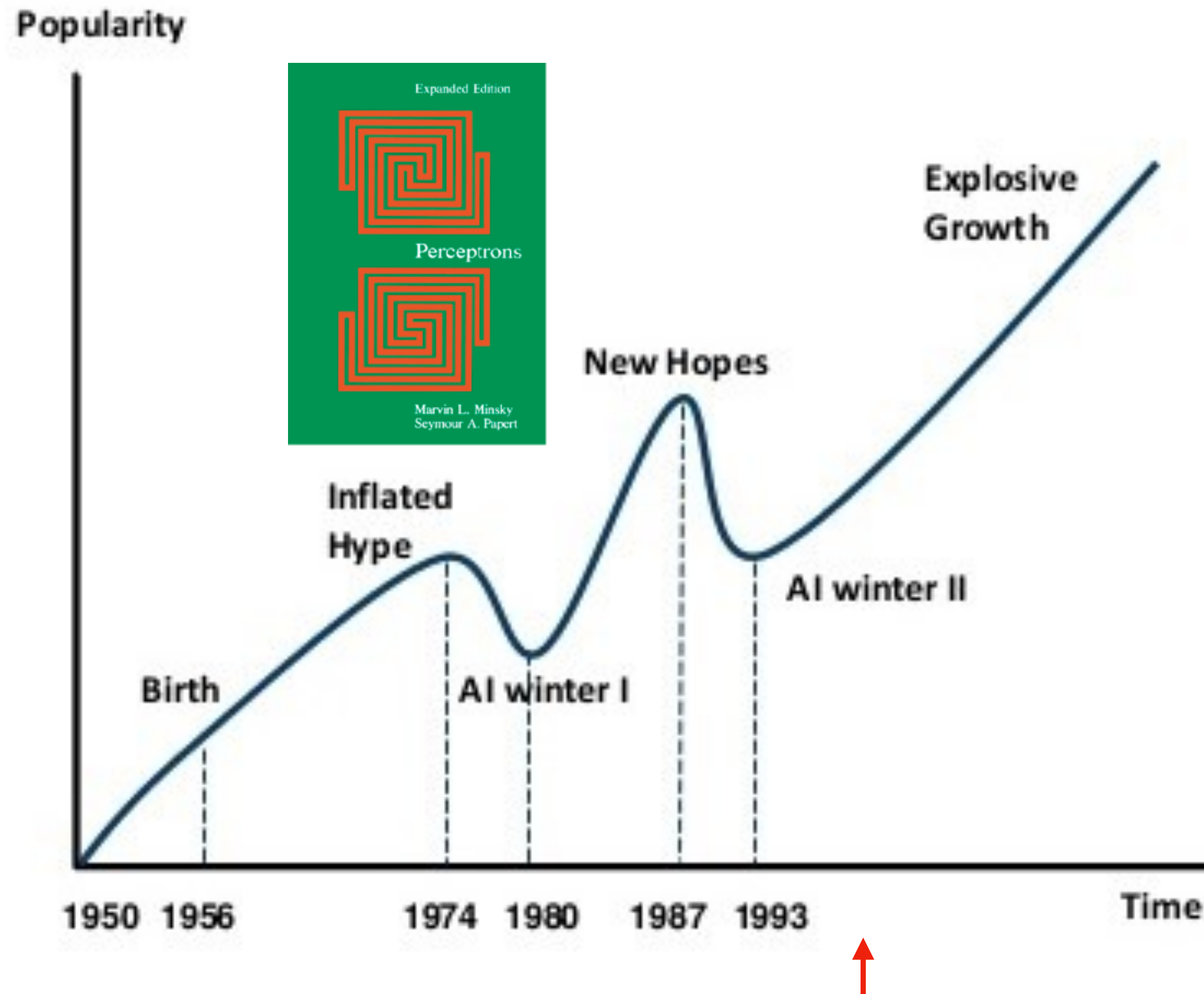
# Timeline of AI



- Skepticism about Perceptrons not being able to solve XOR problems led to the first AI winter

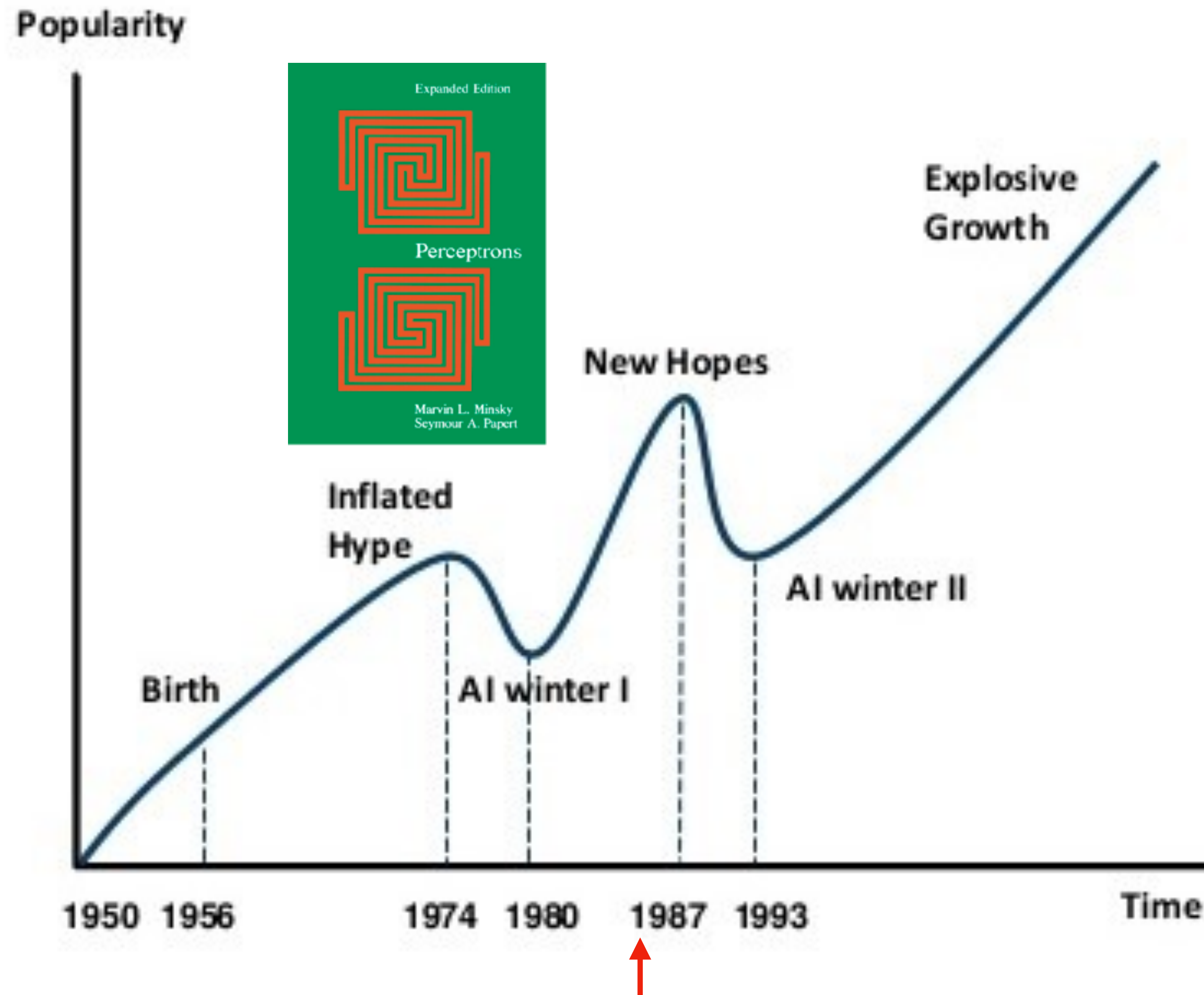


# Timeline of AI



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# Timeline of AI



- Skepticism about Perceptrons not being able to solve XOR problems led to the first AI winter
- It wouldn't be until the deep learning revolution (~2006) that artificial neural networks would experience the same level of popularity
- But what happened in the 1980s when AI was more popular than ever? And why was there a 2nd AI winter?

# Symbolic AI

- **Physical Symbol System hypothesis:**

*“A physical symbol system has the necessary and sufficient means for general intelligent action - Allen Newell and Herbert Simon (1976)”*



Herbert Simon  
& Allen Newell



# Symbolic AI

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- **Symbols** can represent things in the world

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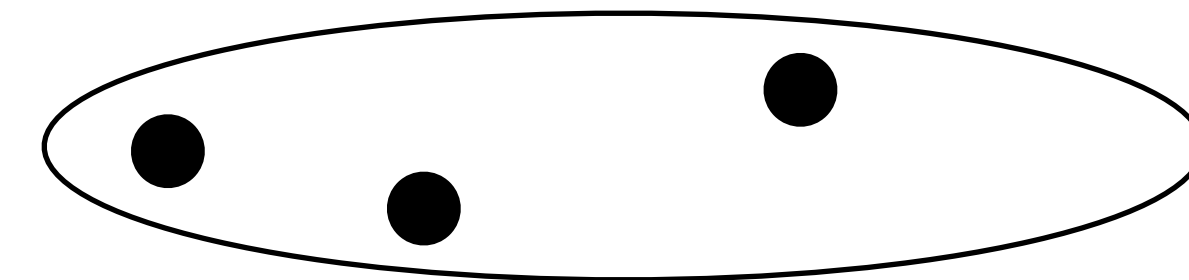
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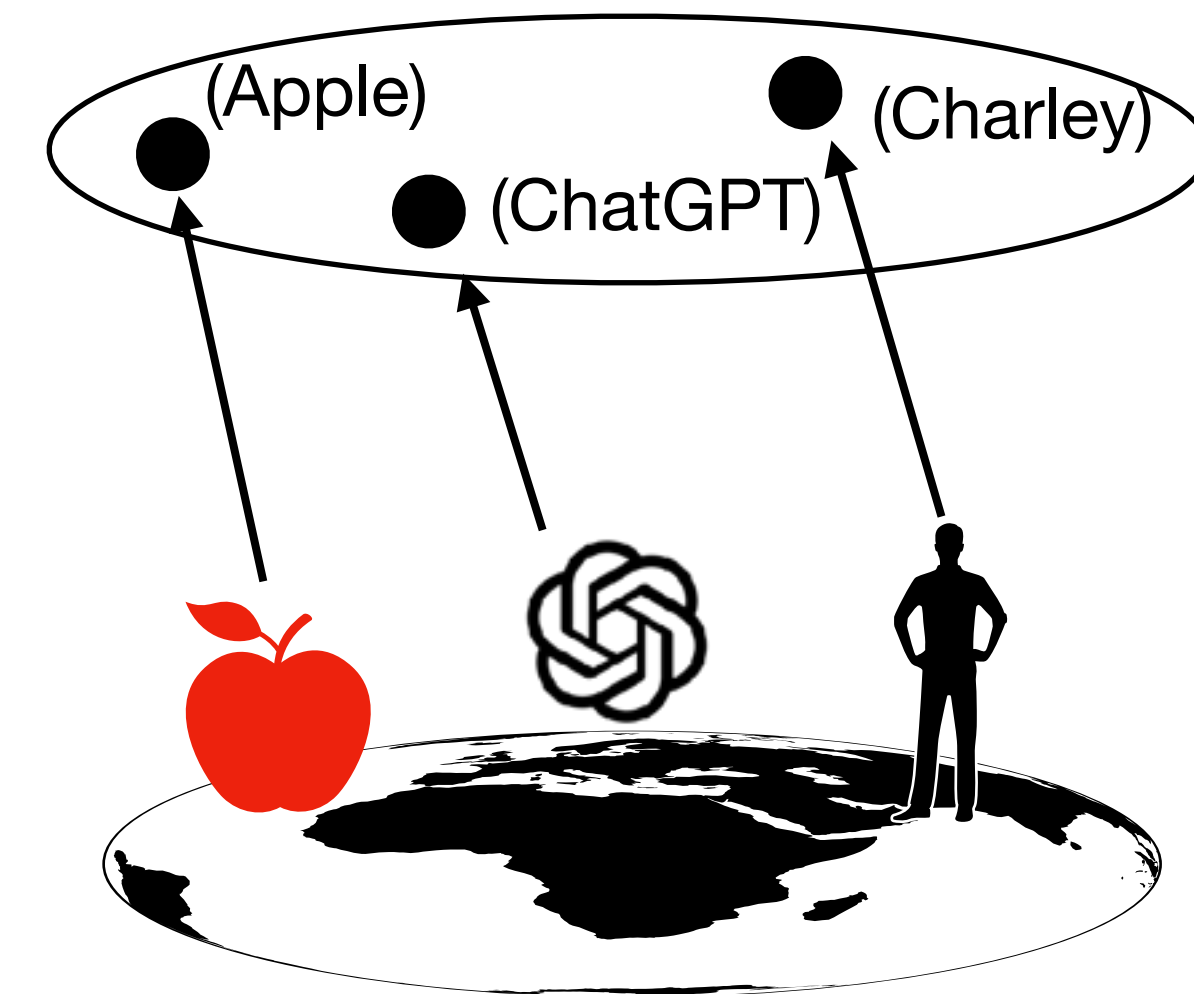
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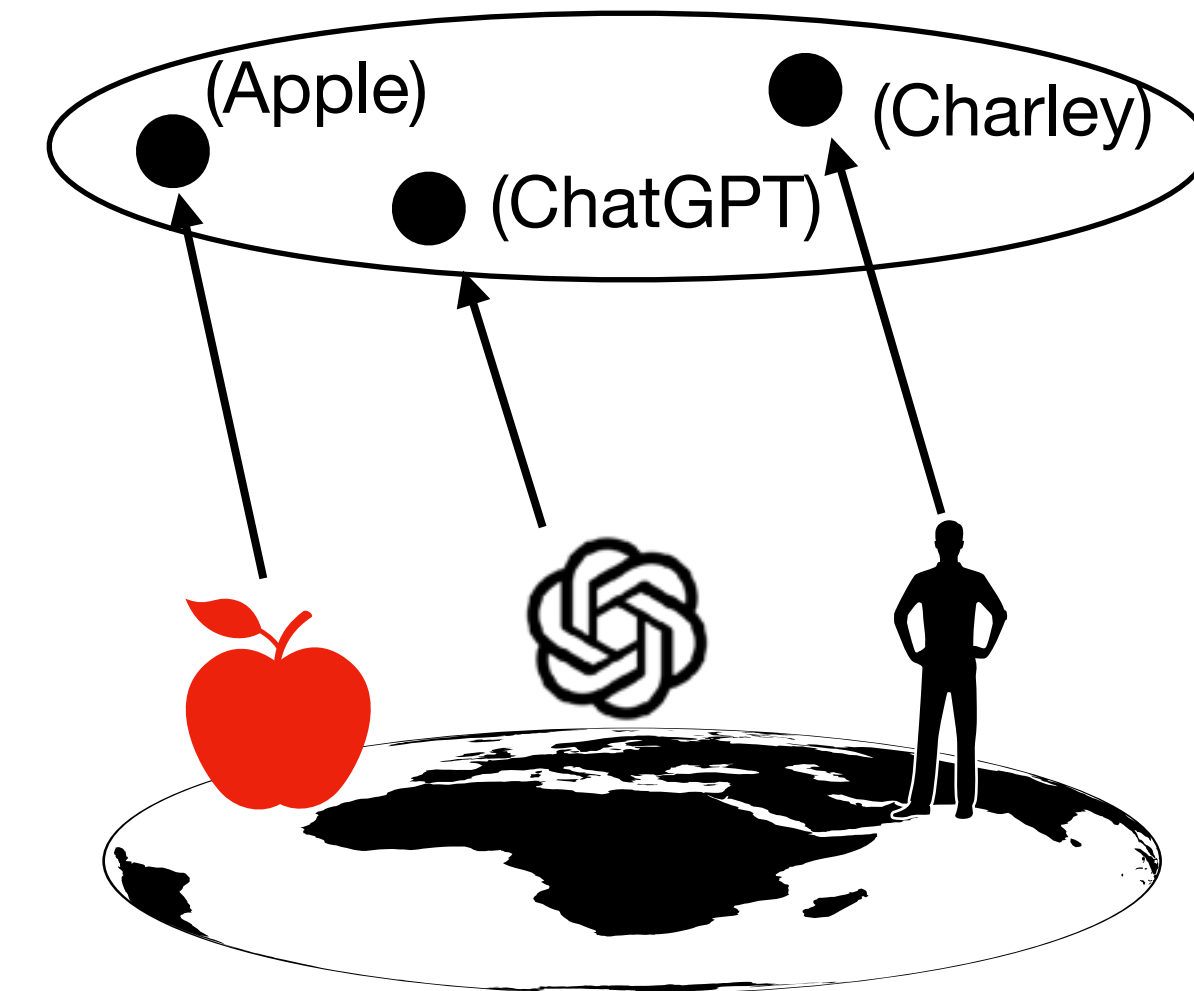
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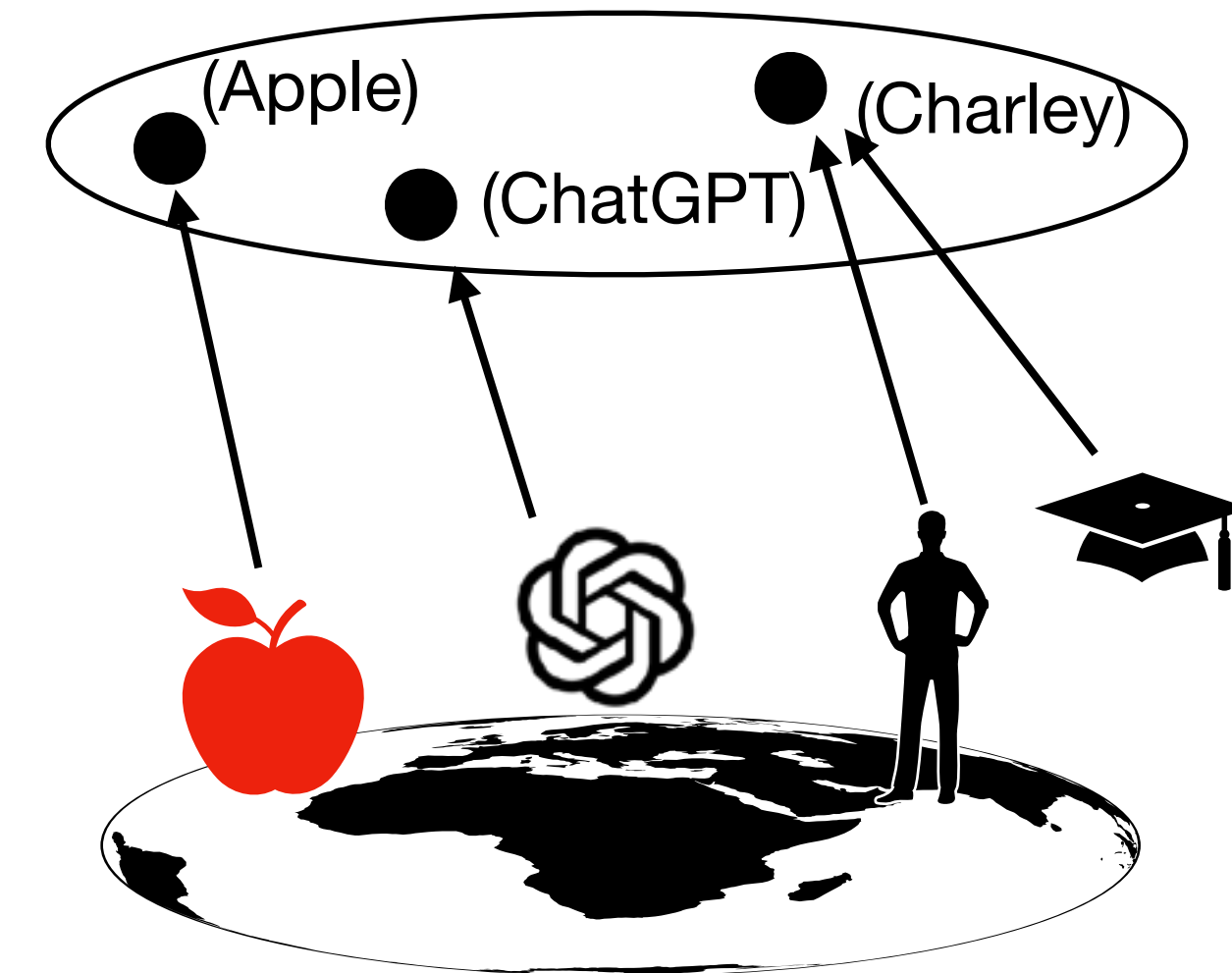
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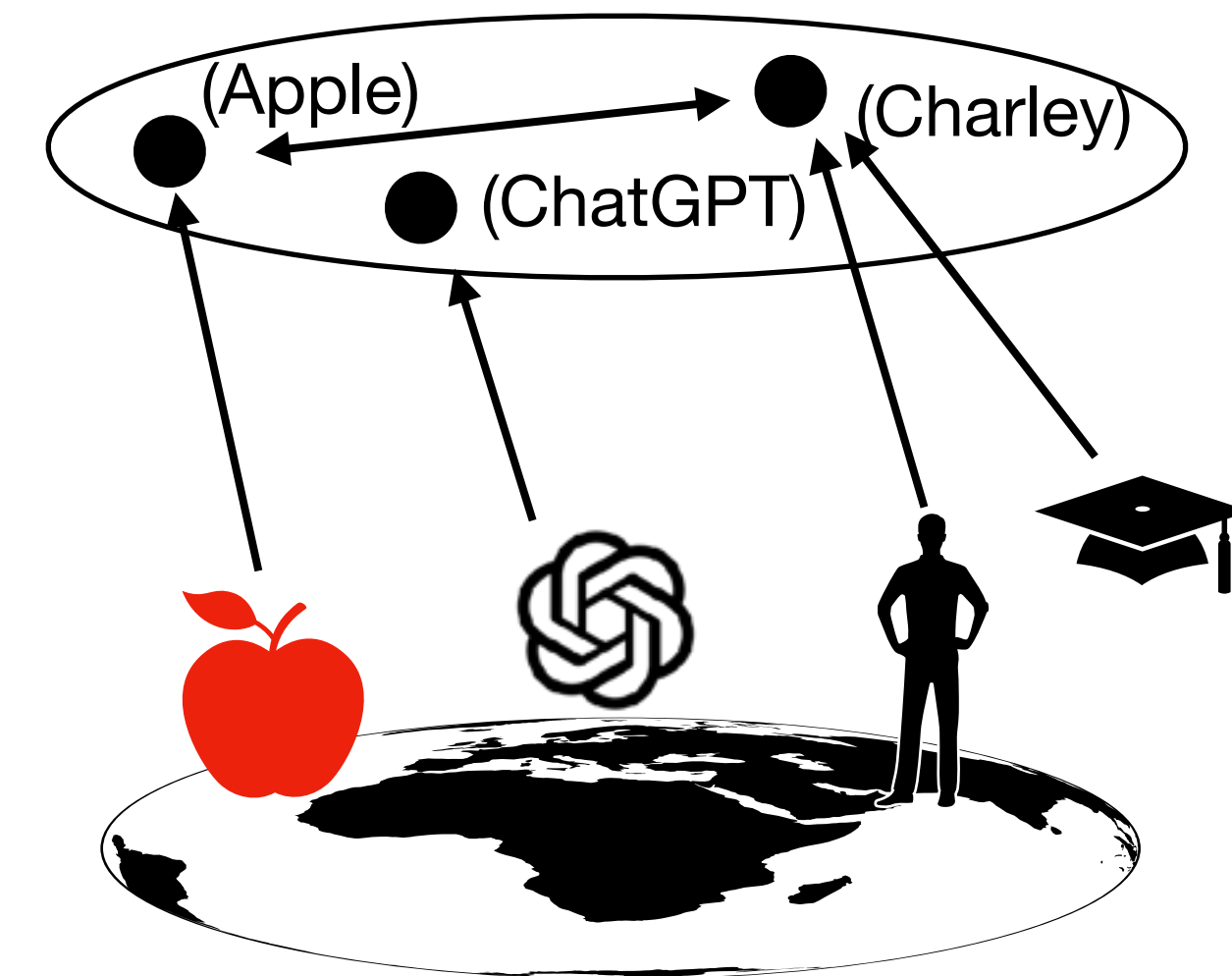
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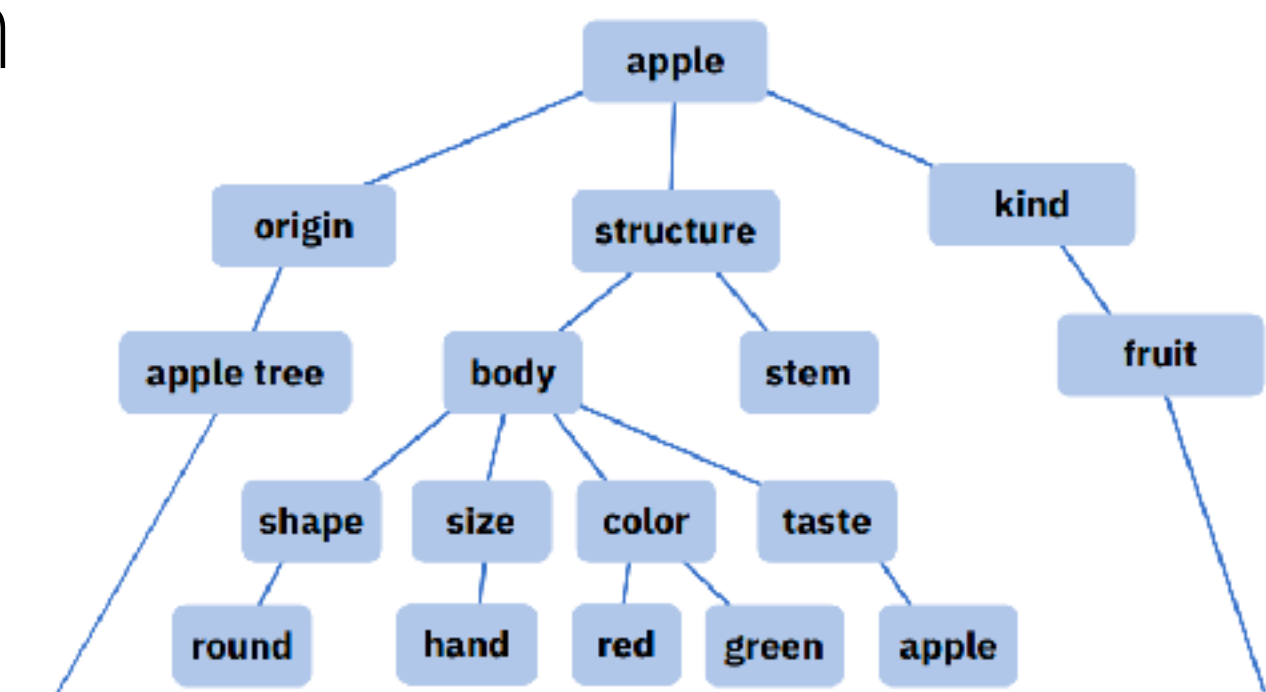
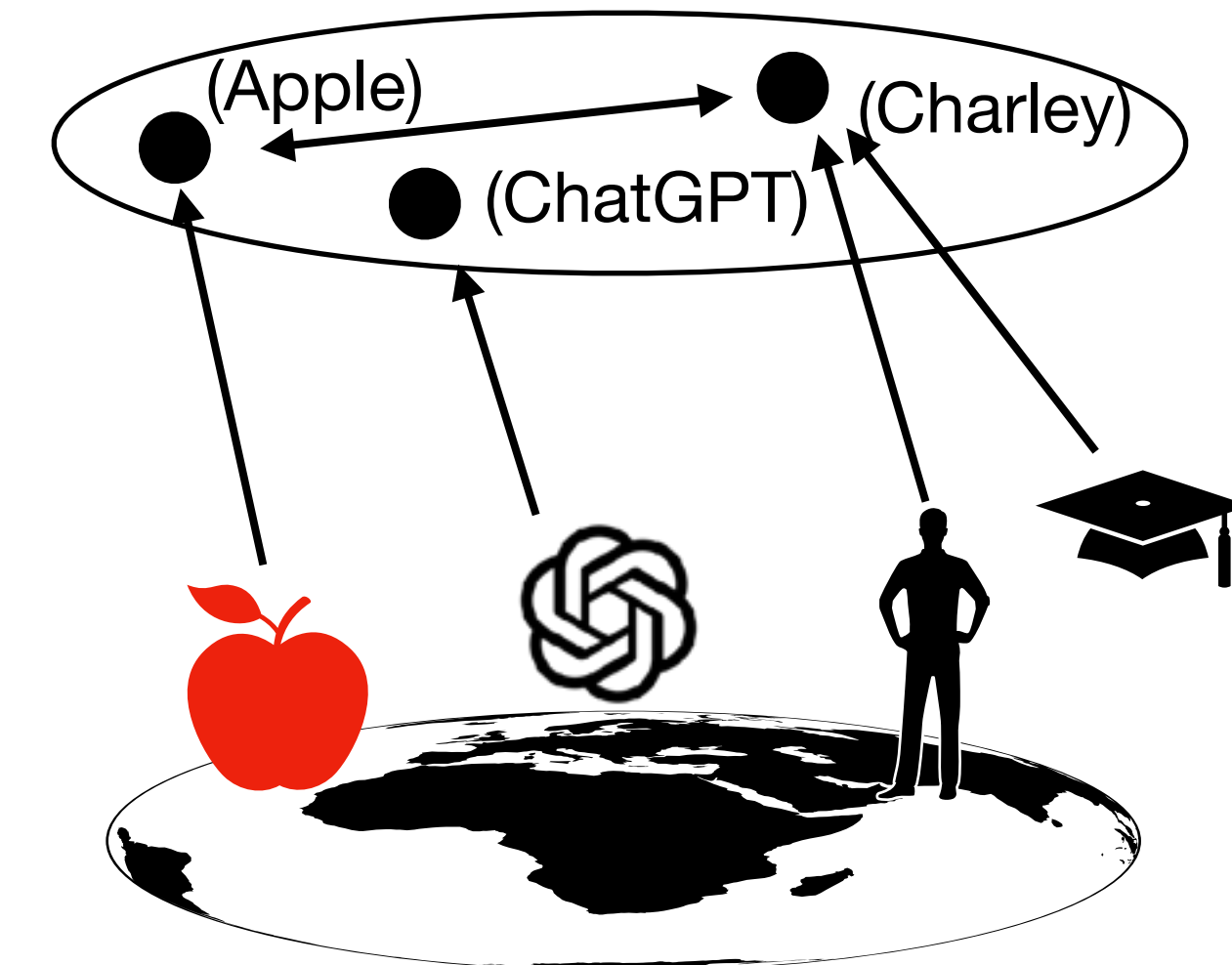
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- By populating a **knowledge base** with symbols and relations, we can use a program to find new propositions (*inference*)

- General Problem Solver (Simon, Shaw, & Newell, 1957)
- Expert systems: popularized in the 1980s as the future of AI



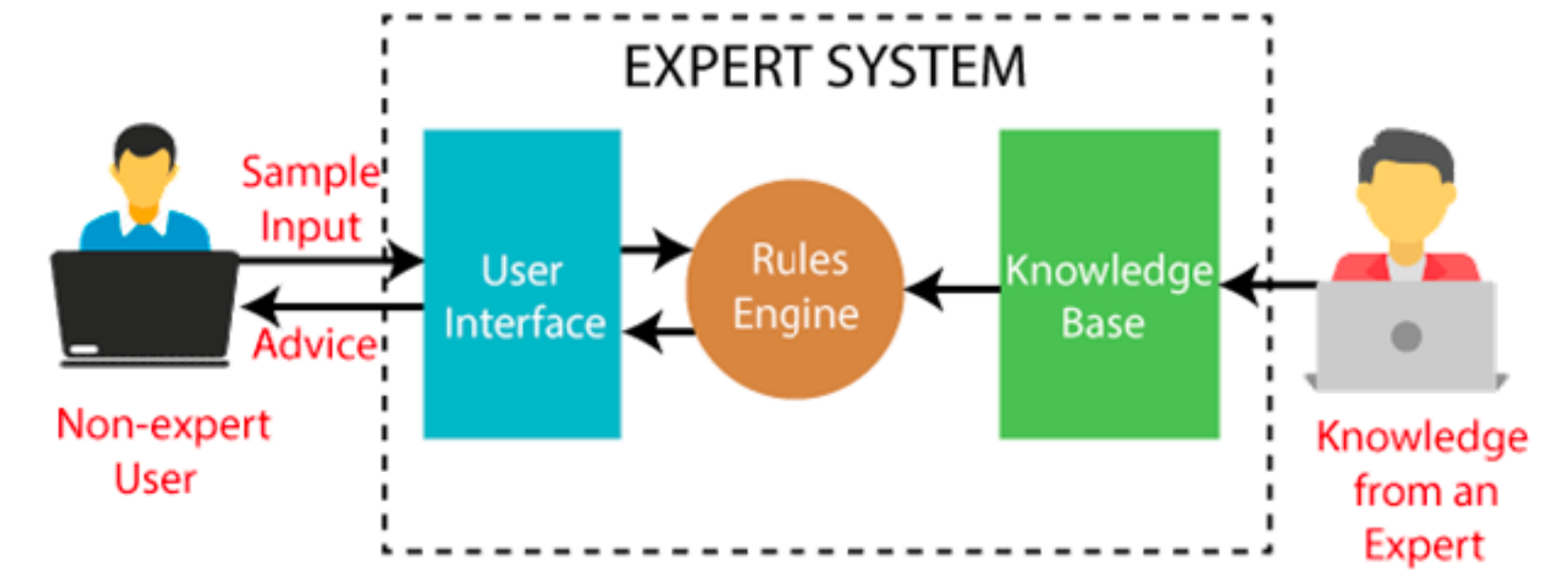
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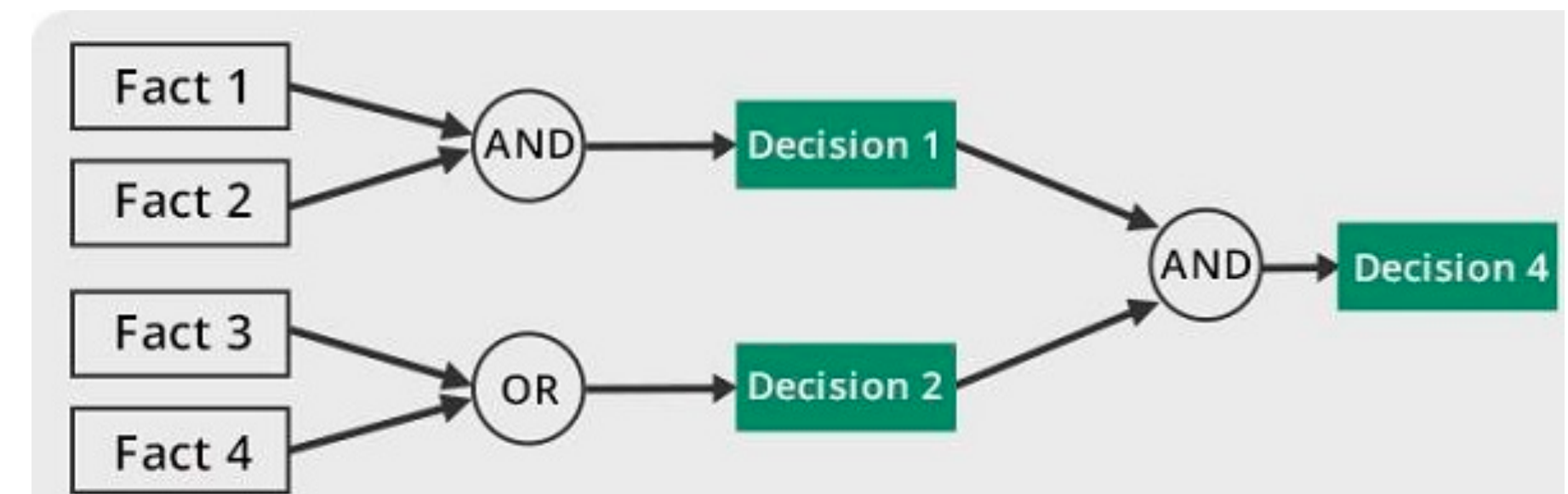
# Expert Systems

- The first truly successful forms of AI, widely applied in medicine, finance, and education
- Expert knowledge is codified in the form of facts and logical rules by a *knowledge engineer*
  - If X then Y
  - If Socrates is a man, then Socrates is mortal
- This forms the basis of an *inference engine*, which can apply known rules/facts to generate new facts (adding to the knowledge base) and resolve rule conflicts
- Two modes for solving problems

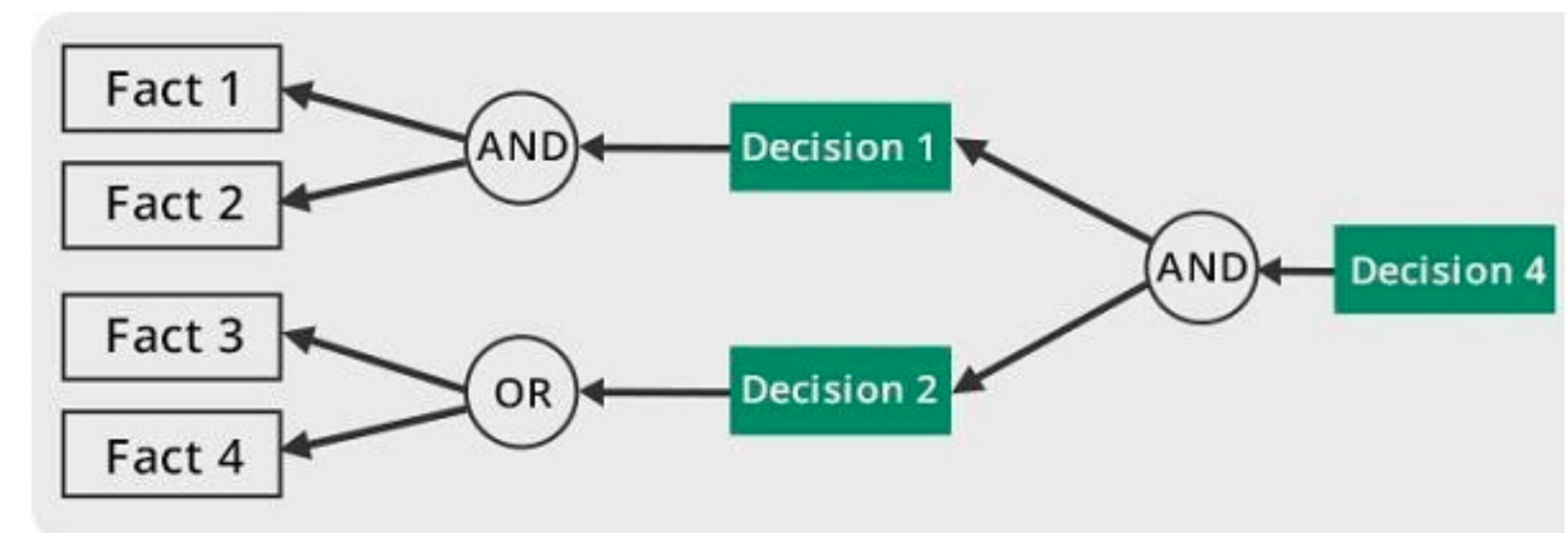
- **Forward chaining:** What happens next? **not on the exam**
  - Apply rules and facts to arrive at logical conclusions about outcomes
- **Backwards chaining:** Why did it happen?
  - Starting from a desired outcome, figure out the set of antecedents that can aid in arriving at that outcome



## Forward chaining

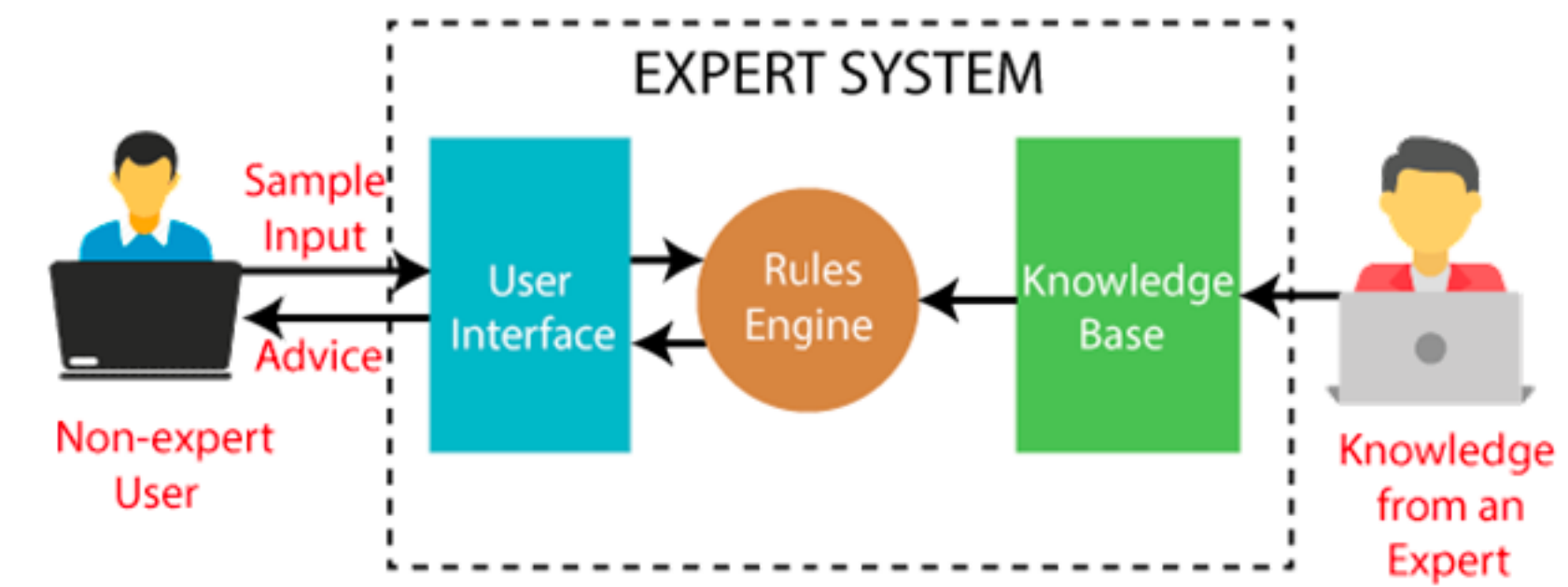


## Backward chaining





# Strengths and Limitations



## Strengths

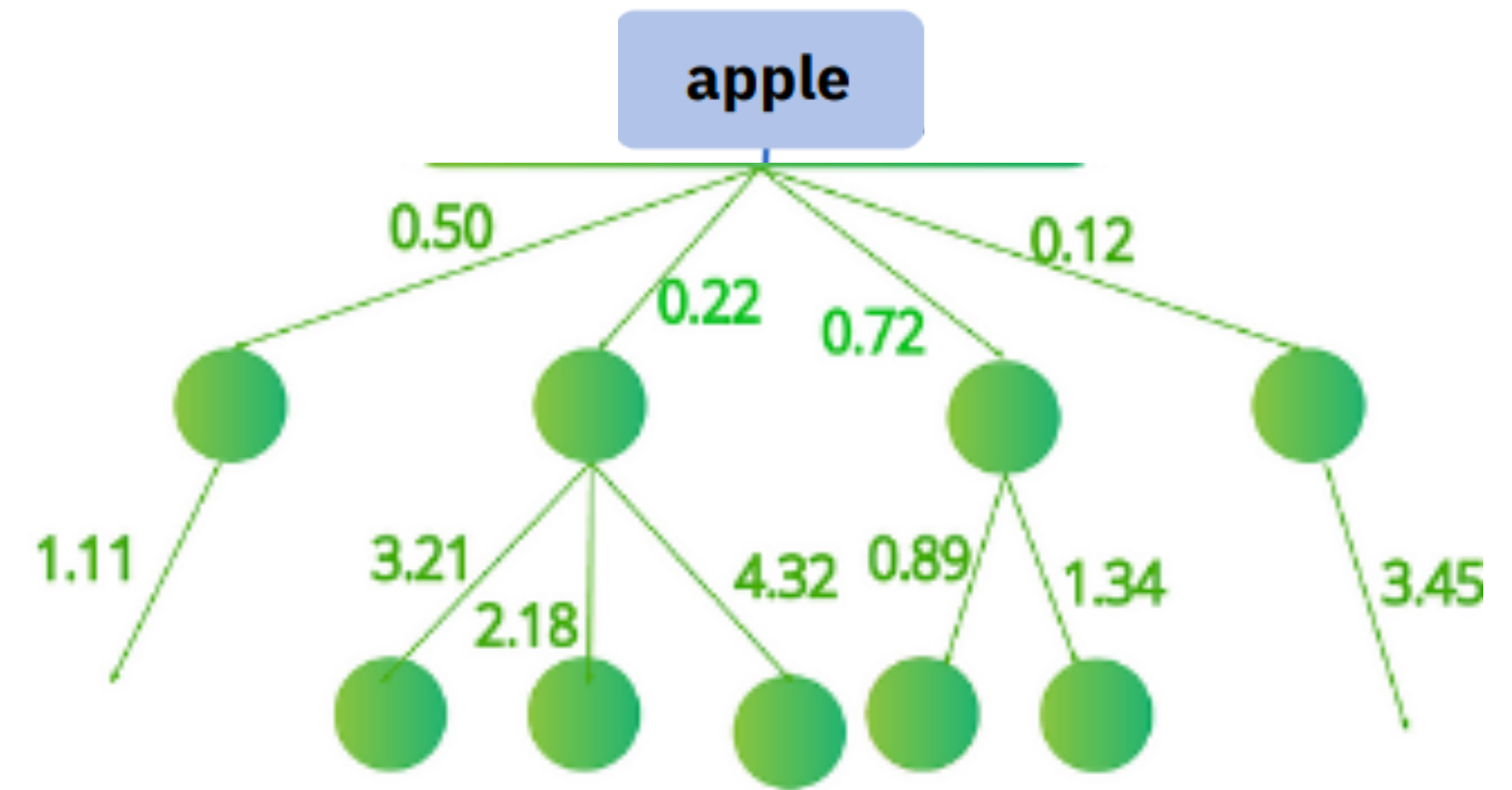
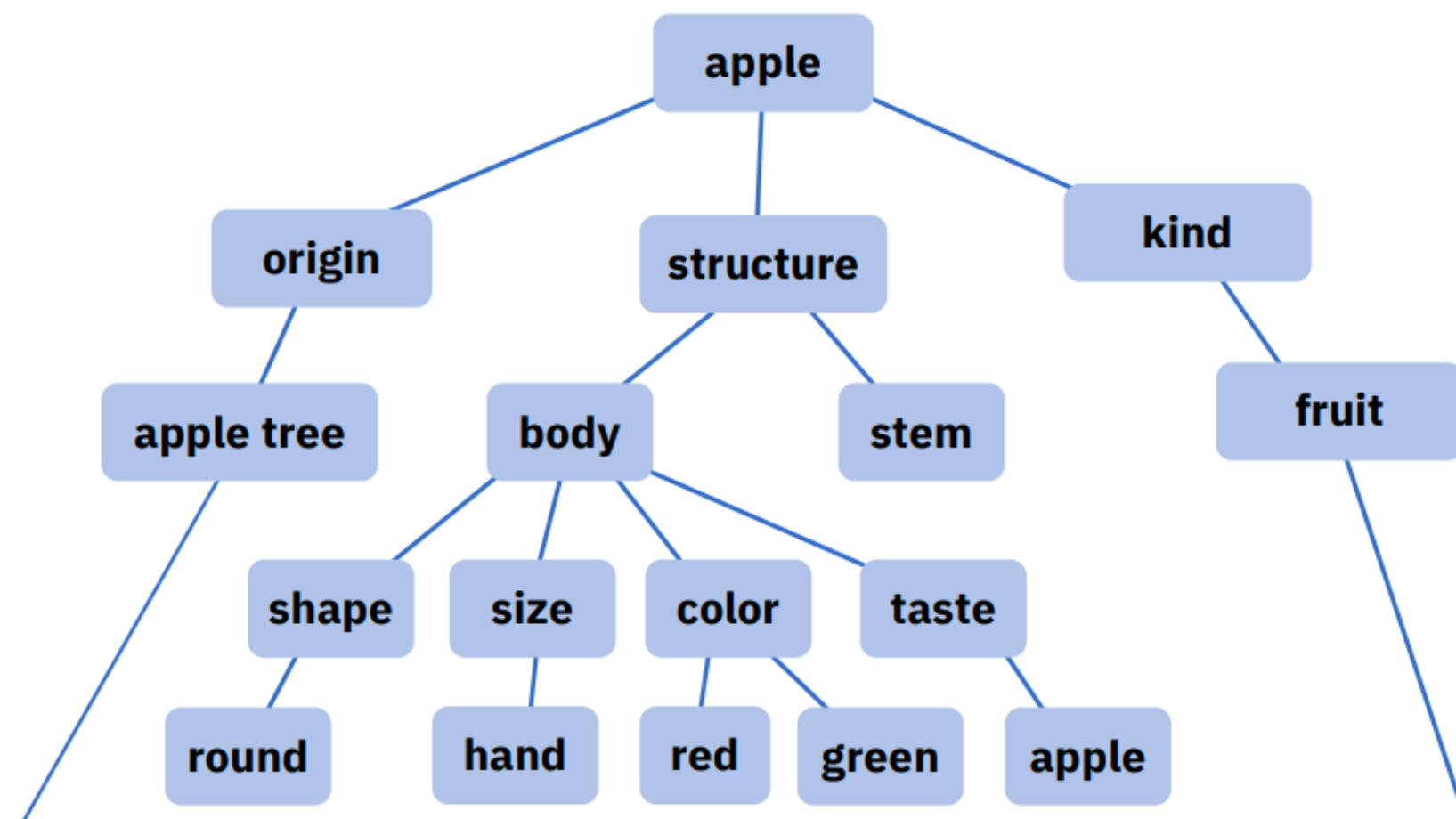
- Knowledge is explicit rather than implicit (e.g., neural networks), allowing for interpretability
- Applying rules can be very fast and solutions were generated in real-time
- Rules offer rapid generalization, with a single instance
- Decisions are interpretable by following logic
- No hallucinations!

## Limitations

- Cannot learn by itself!
- Require knowledge engineers to codify rules, with high maintenance and development costs
- Limited generalization to new situations, where existing rules don't apply exactly
- If-Then statements cannot capture all relationships without massive scaling problems



# Symbolic vs. sub-symbolic AI



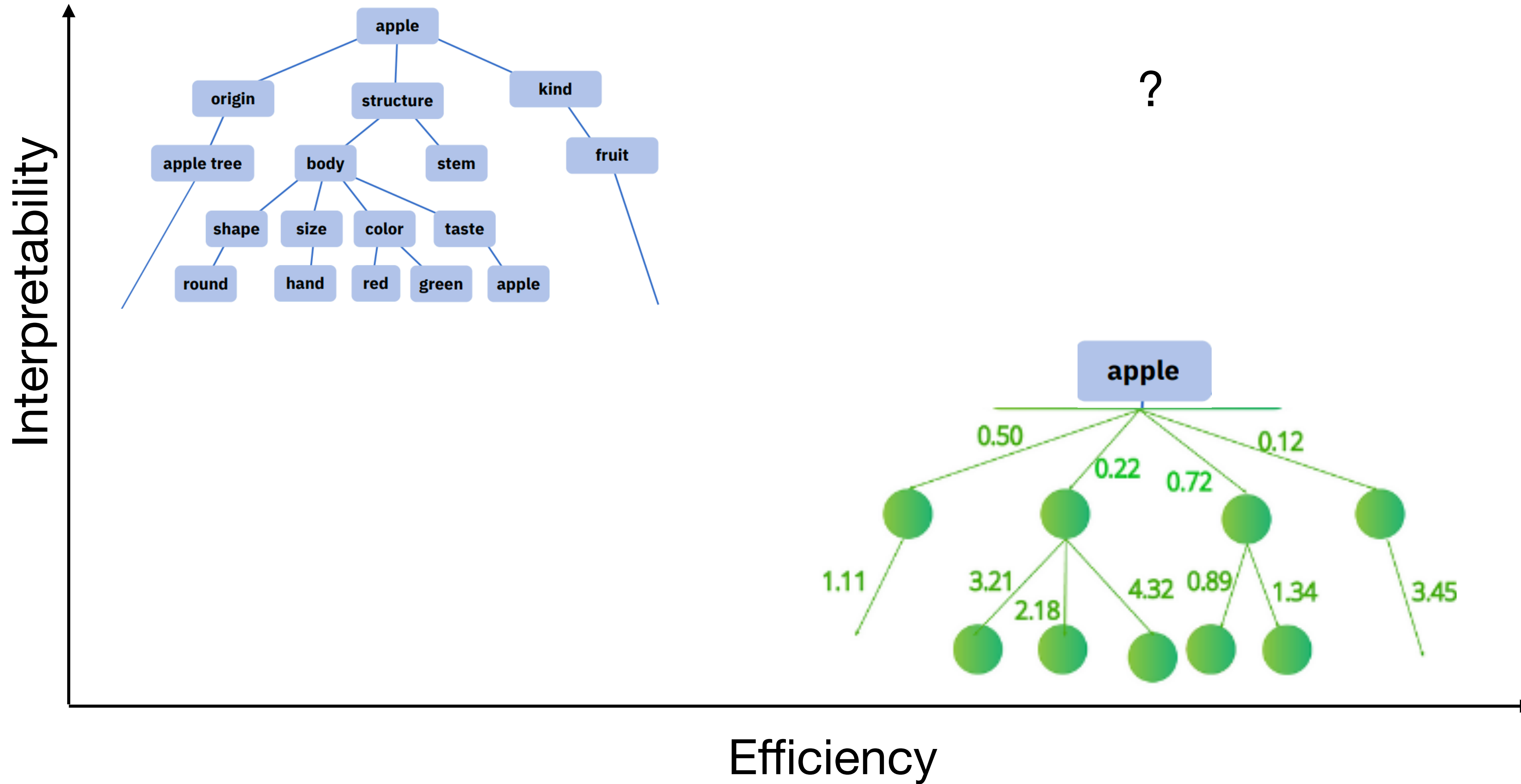
## Symbolic AI

- Symbols, rules, and structured representations
- **“Language of thought”** (LoT) hypothesis (Fodor, 1975): concepts/knowledge represented by a language-like system
- Compositionality: symbols and rules can be combined to produce new representations
- Extracting symbolic representations and search over compositional hypothesis spaces is difficult

## Sub-symbolic AI

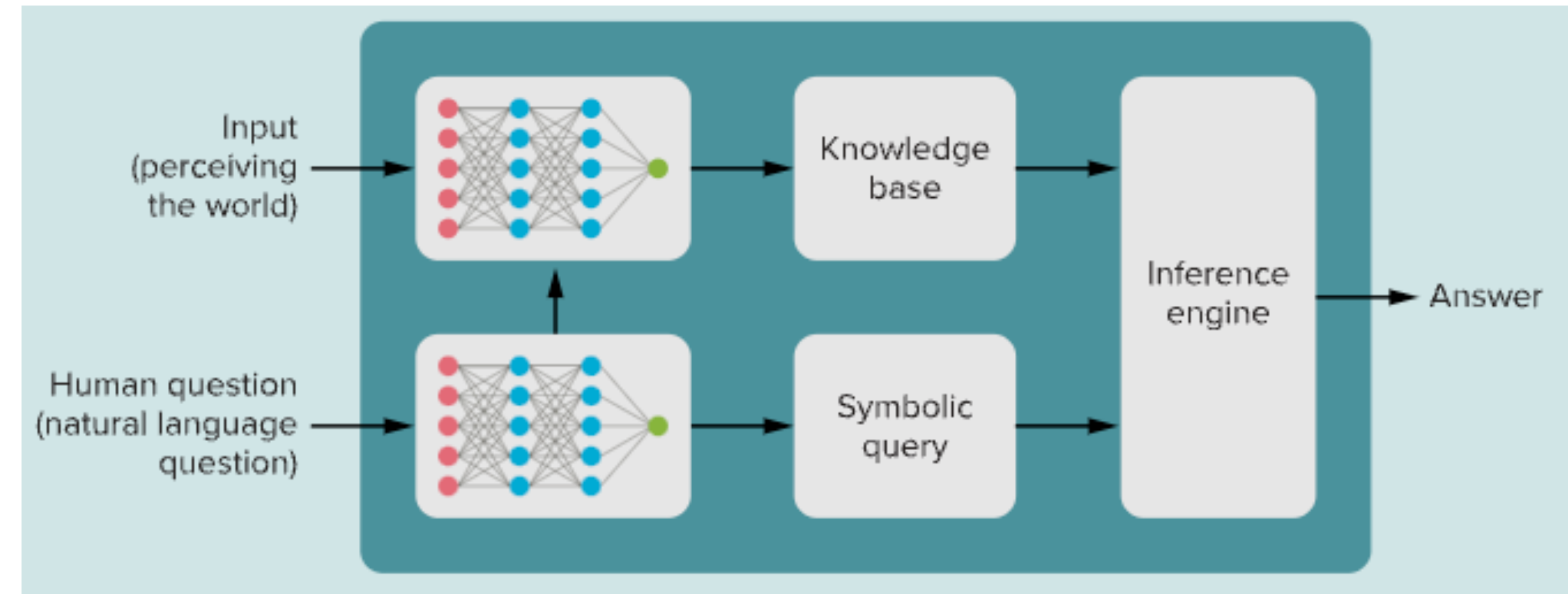
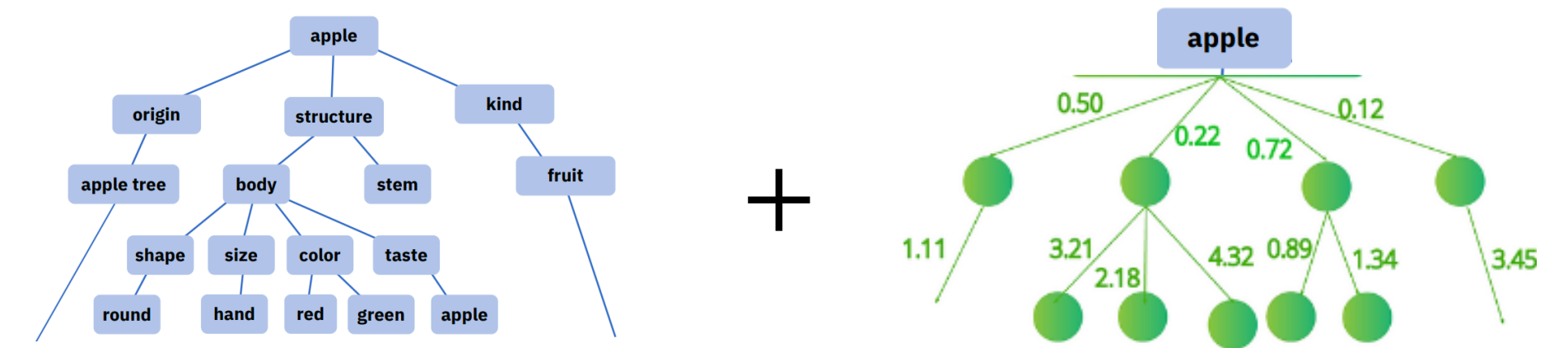
- Representations encoded through connection weights
- No explicit representation of concepts or knowledge, but distributed throughout the network
- Efficiency: knowledge can be implicitly learned by capturing statistical patterns
- Interpretation of representations and behavior is difficult

# Symbolic vs. sub-symbolic AI



# Neurosymbolic AI

- Neurosymbolic AI aims to combine symbolic and subsymbolic approaches to get the best of both worlds
- Modern AI assistants (e.g., Siri, Google, Alexa) are essentially expert systems with ANN voice recognition and text-to-speech



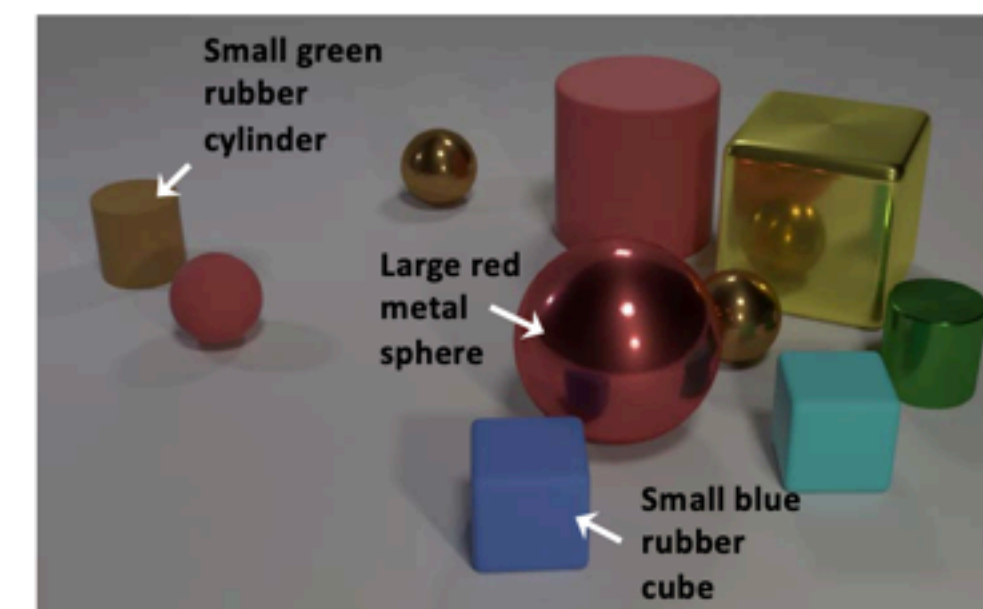
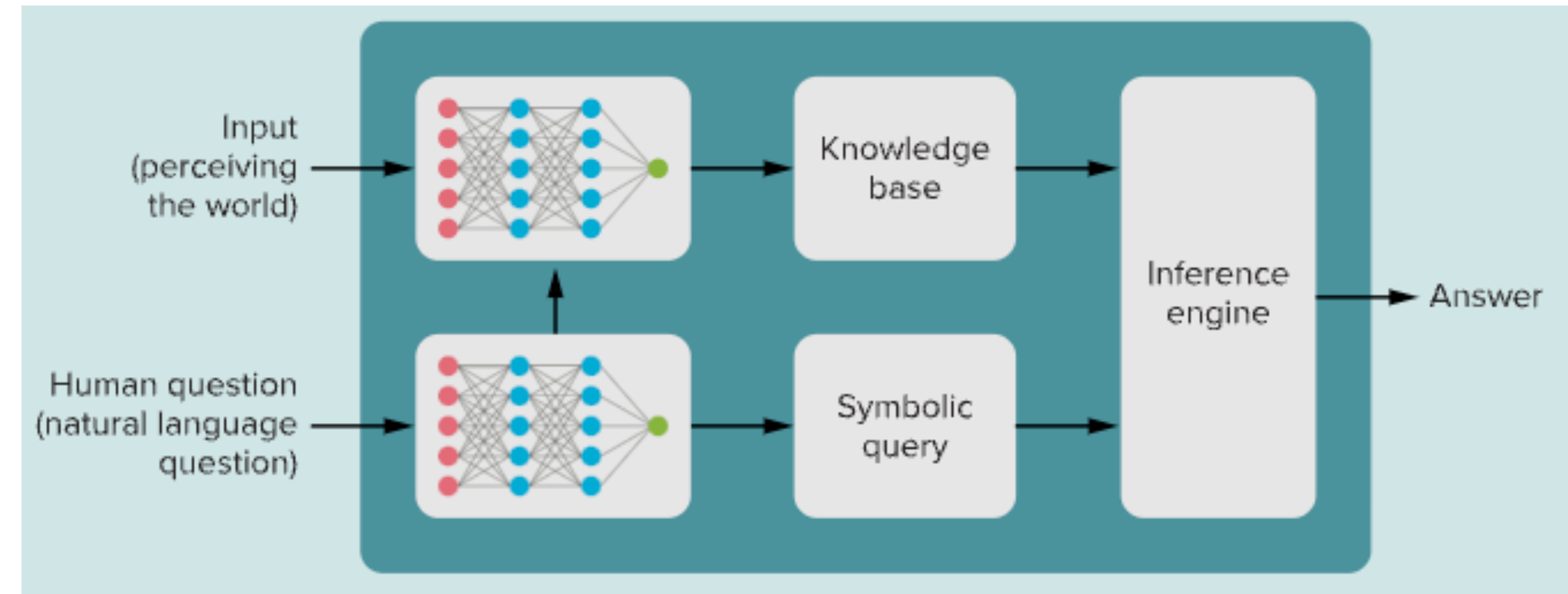
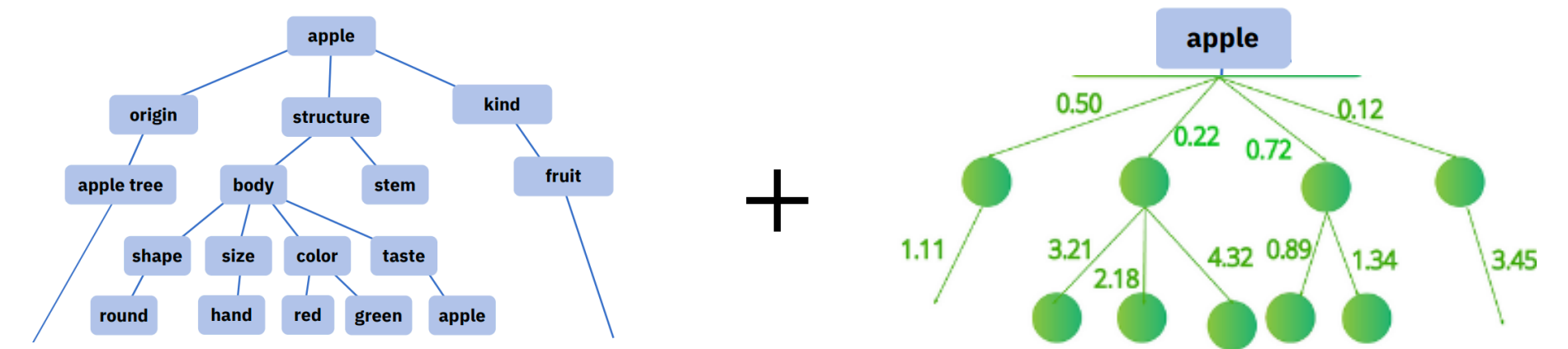


# Neurosymbolic AI

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- Current challenges:
  - Learning the knowledge base through data
  - Relating messy real-world data to neat (and limited) symbols/relations in a knowledge base



Yi et al., (2018)

**Question:** *Are there an equal number of large things and metal spheres?*

**Program:** `equal_number(count(filter_size(Scene, Large)), count(filter_material(filter_shape(Scene, Sphere), Metal)))`

**Answer:** *Yes*

**How can symbolic knowledge be learned from data?**

# How can symbolic knowledge be learned from data?

One-shot generalization



Lake et al., (*Science* 2015)



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Parsing into parts and relations



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Generalization from related concepts



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## Program Induction

*the process of inferring **rules or instructions** that generate an observed pattern of data*



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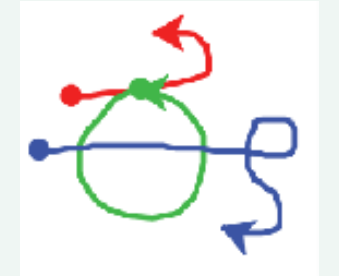
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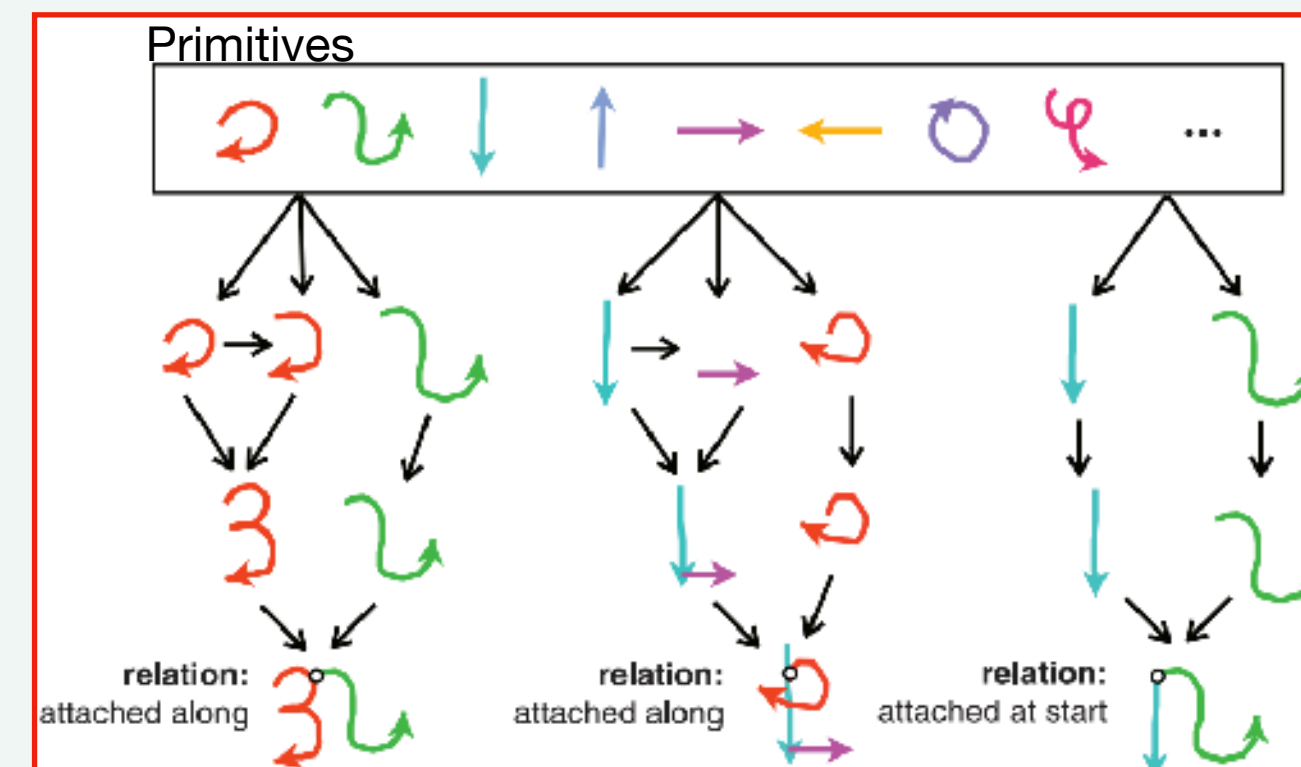
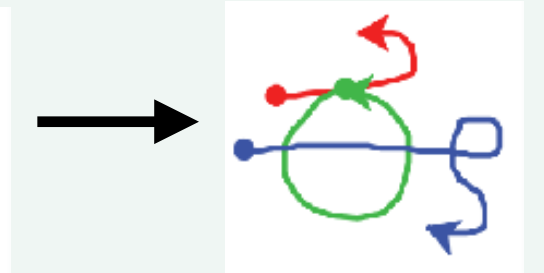
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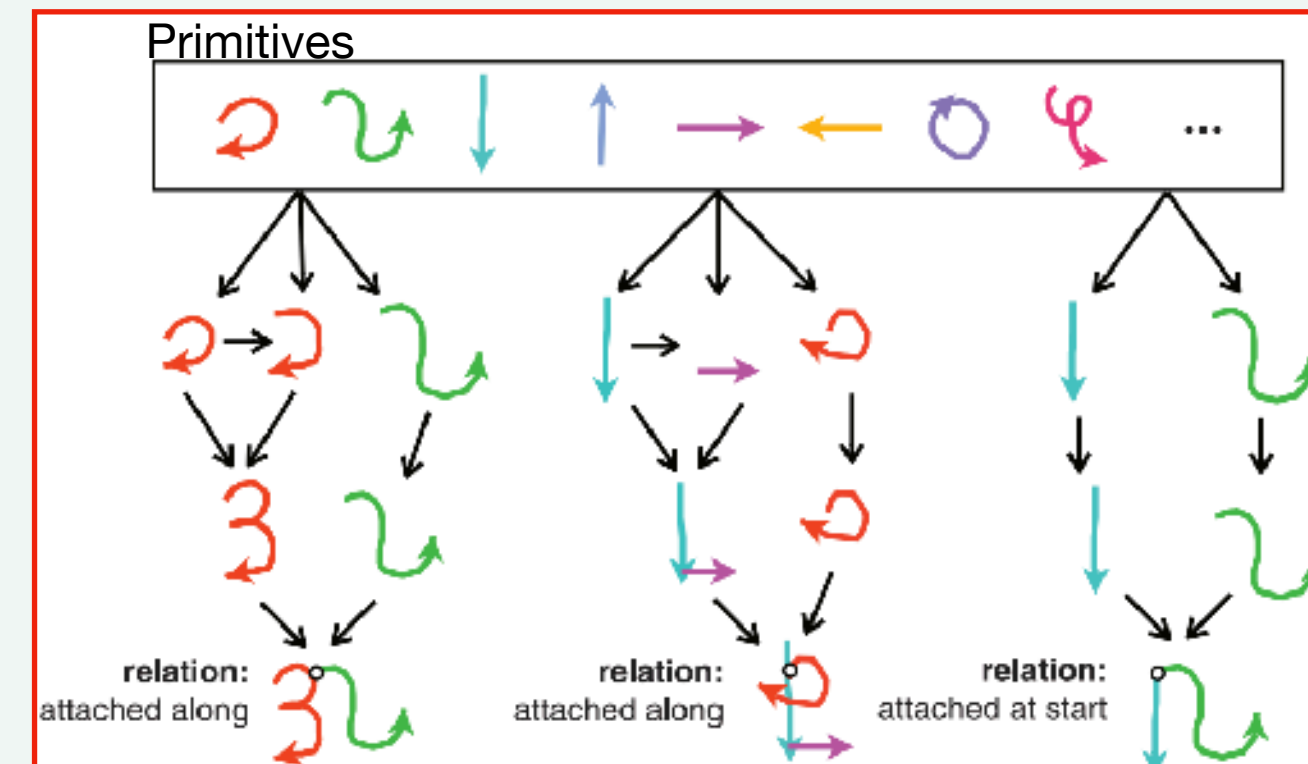
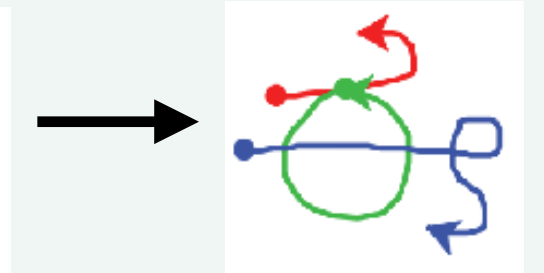
$$P(\pi | \mathcal{D}) \propto P(\mathcal{D} | \pi)P(\pi)$$

the process of inferring **rules or instructions** that generate an observed pattern of data

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Program  $\pi$



Lake et al., (Science 2015)

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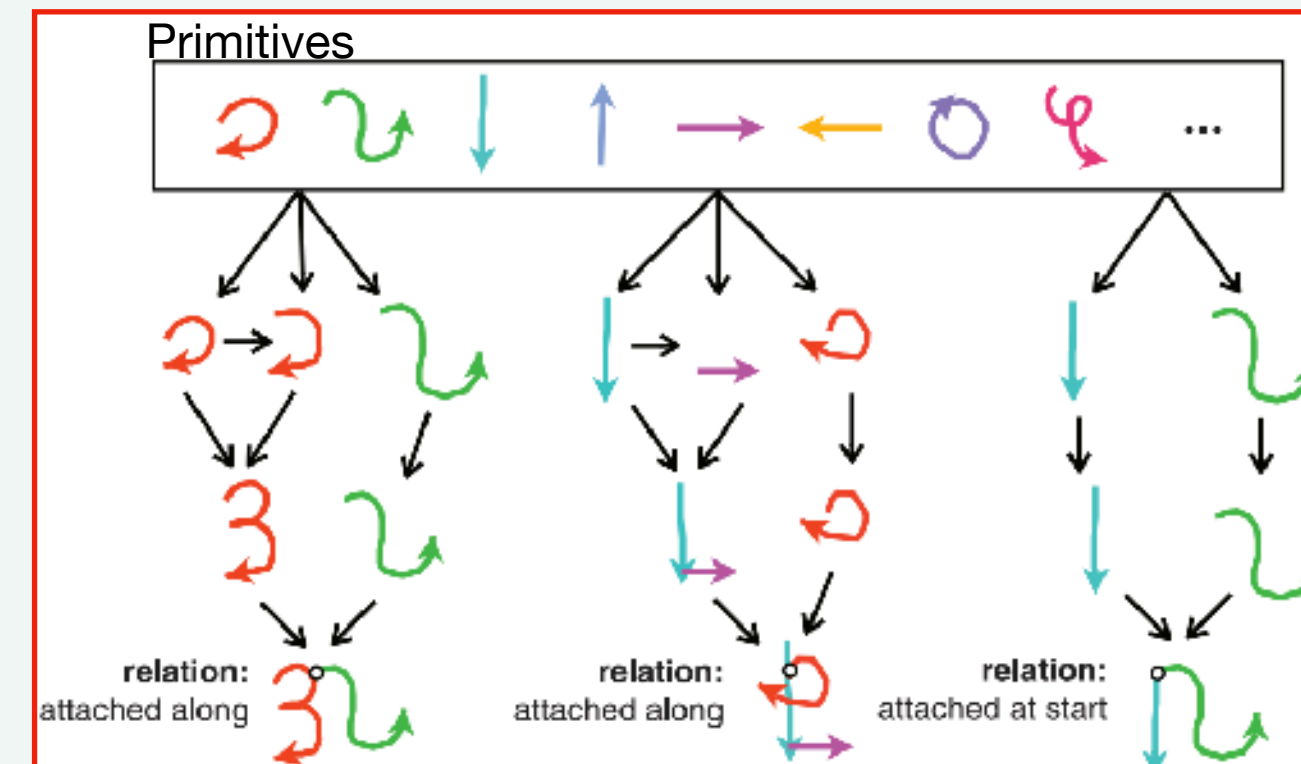
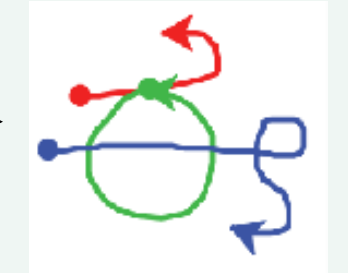
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Program  $\pi$



```

procedure GENERATE TYPE
   $\kappa \leftarrow P(\kappa)$   $\triangleright$  Sample number of parts
  for  $i = 1 \dots \kappa$  do
     $n_i \leftarrow P(n_i | \kappa)$   $\triangleright$  Sample number of sub-parts
    for  $j = 1 \dots n_i$  do
       $s_{ij} \leftarrow P(s_{ij} | s_{i(j-1)})$   $\triangleright$  Sample sub-part sequence
    end for
     $R_i \leftarrow P(R_i | S_1, \dots, S_{i-1})$   $\triangleright$  Sample relation
  end for
   $\psi \leftarrow \{\kappa, R, S\}$ 
  return @GENERATE TOKEN( $\psi$ )  $\triangleright$  Return program
  
```

Lake et al., (Science 2015)



# How can symbolic knowledge be learned from data?

One-shot generalization



Parsing into parts and relations



Generalization from related concepts



## Program Induction

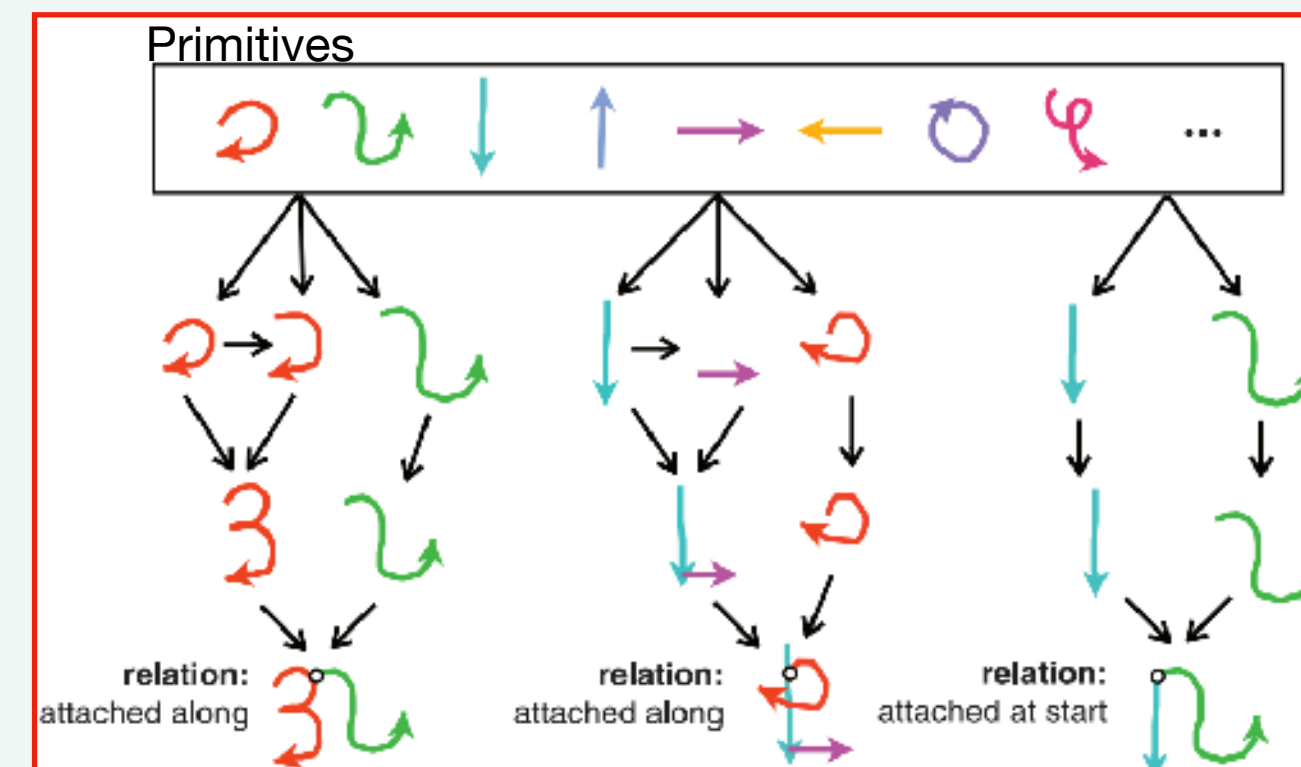
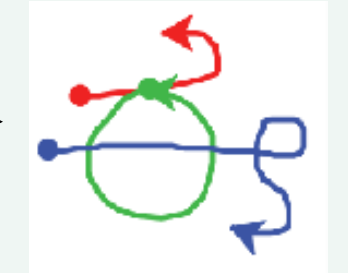
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Program  $\pi$



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  end for
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  return @GENERATETOKEN( $\psi$ ) ▷ Return program
  
```

Lake et al., (Science 2015)

### List Processing

#### Sum List

[1 2 3] → 6  
[4 6 8 1] → 17

#### Double

[1 2 3] → [2 4 6]  
[4 5 1] → [8 10 2]

#### Check Evens

[0 2 3] → [T T F]  
[2 9 6] → [T F T]

### Text Editing

#### Abbreviate

Allen Newell → A.N.  
Herb Simon → H.S.

#### Drop Last Three

shrdlu → shr  
shakey → sha

#### Extract

a b (c) → c  
a (bee) see → see

### Regexes

#### Phone numbers

(555) 867-5309  
(650) 555-2368

#### Currency

\$100.25  
\$4.50

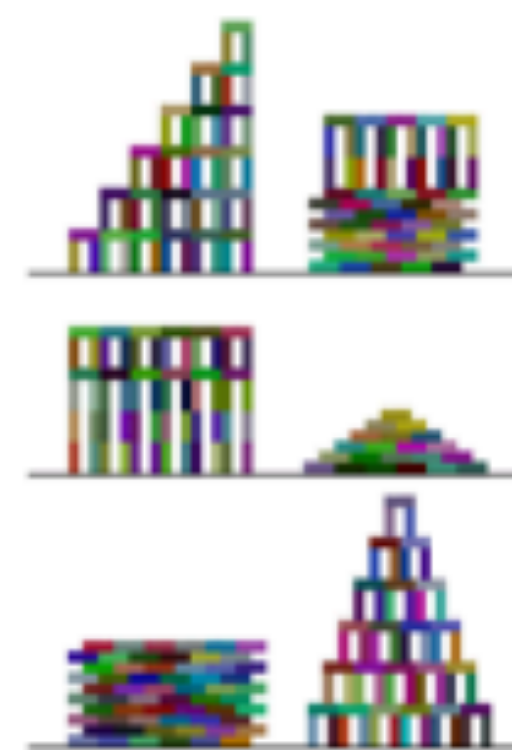
#### Dates

Y1775/0704  
Y2000/0101

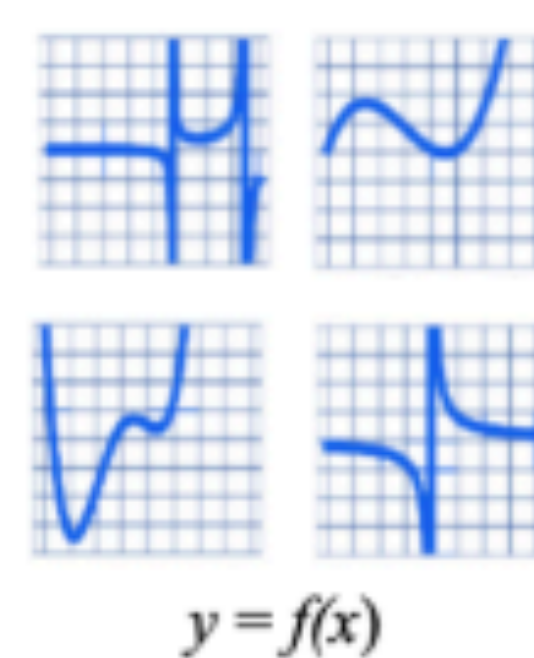
### LOGO Graphics



### Block Towers



### Symbolic Regression



### Recursive Programming

#### Filter Red

[red blue] → [blue]  
[red blue green] → [blue green]  
[red blue green red] → [blue green]

#### Length

[red blue] → 4  
[red blue green] → 6  
[red blue] → 3

### Physical Laws

$$\vec{a} = \frac{1}{m} \sum_i \vec{F}_i$$

$$\vec{F} \propto \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

$$R_{\text{total}} = \left( \sum_i \frac{1}{R_i} \right)^{-1}$$



# How can symbolic knowledge be learned from data?

One-shot generalization



Parsing into parts and relations



Generalization from related concepts



## Program Induction

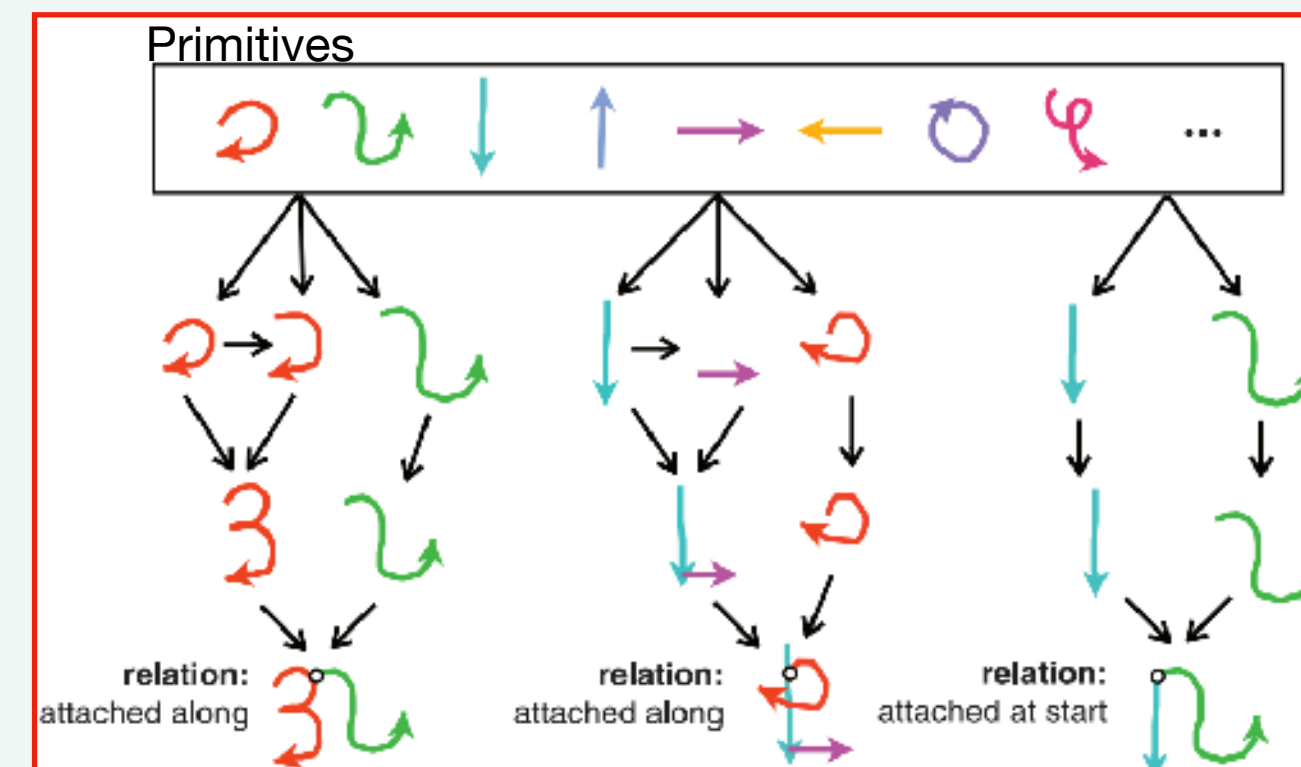
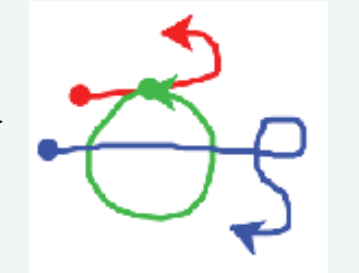
the process of inferring **rules or instructions** that generate an observed pattern of data

$$P(\pi | \mathcal{D}) \propto P(\mathcal{D} | \pi)P(\pi)$$

Data  $\mathcal{D}$



Program  $\pi$



```

procedure GENERATETYPE
   $\kappa \leftarrow P(\kappa)$  ▷ Sample number of parts
  for  $i = 1 \dots \kappa$  do
     $n_i \leftarrow P(n_i | \kappa)$  ▷ Sample number of sub-parts
    for  $j = 1 \dots n_i$  do
       $s_{ij} \leftarrow P(s_{ij} | s_{i(j-1)})$  ▷ Sample sub-part sequence
    end for
     $R_i \leftarrow P(R_i | S_1, \dots, S_{i-1})$  ▷ Sample relation
  end for
   $\psi \leftarrow \{\kappa, R, S\}$ 
  return @GENERATETOKEN( $\psi$ ) ▷ Return program
  
```

Lake et al., (Science 2015)

### List Processing

#### Sum List

[1 2 3] → 6  
[4 6 8 1] → 17

#### Double

[1 2 3] → [2 4 6]  
[4 5 1] → [8 10 2]

#### Check Evens

[0 2 3] → [T T F]  
[2 9 6] → [T F T]

### Text Editing

#### Abbreviate

Allen Newell → A.N.  
Herb Simon → H.S.

#### Drop Last Three

shrdlu → shr  
shakey → sha

#### Extract

a b (c) → c  
a (bee) see → see

### Regexes

#### Phone numbers

(555) 867-5309  
(650) 555-2368

#### Currency

\$100.25  
\$4.50

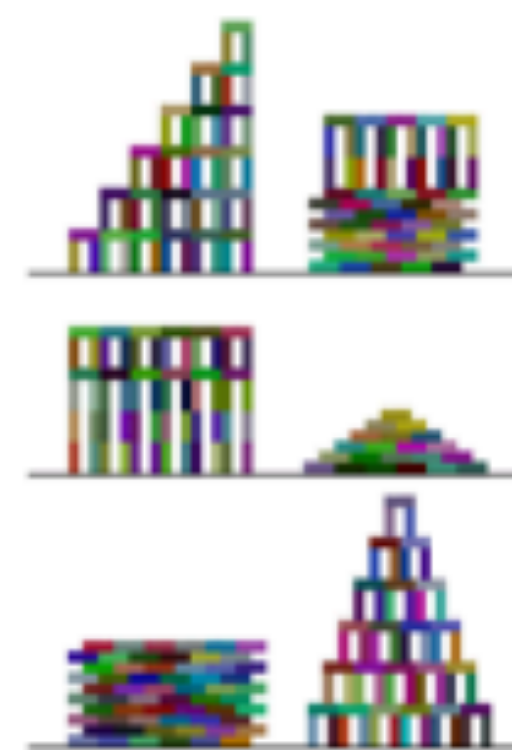
#### Dates

Y1775/0704  
Y2000/0101

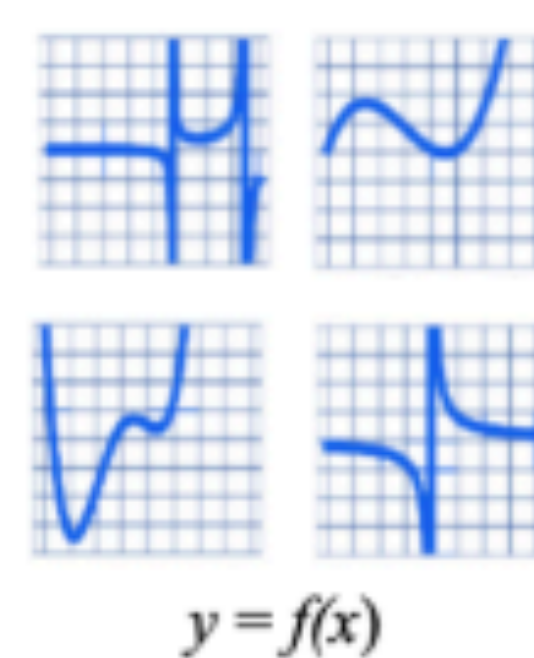
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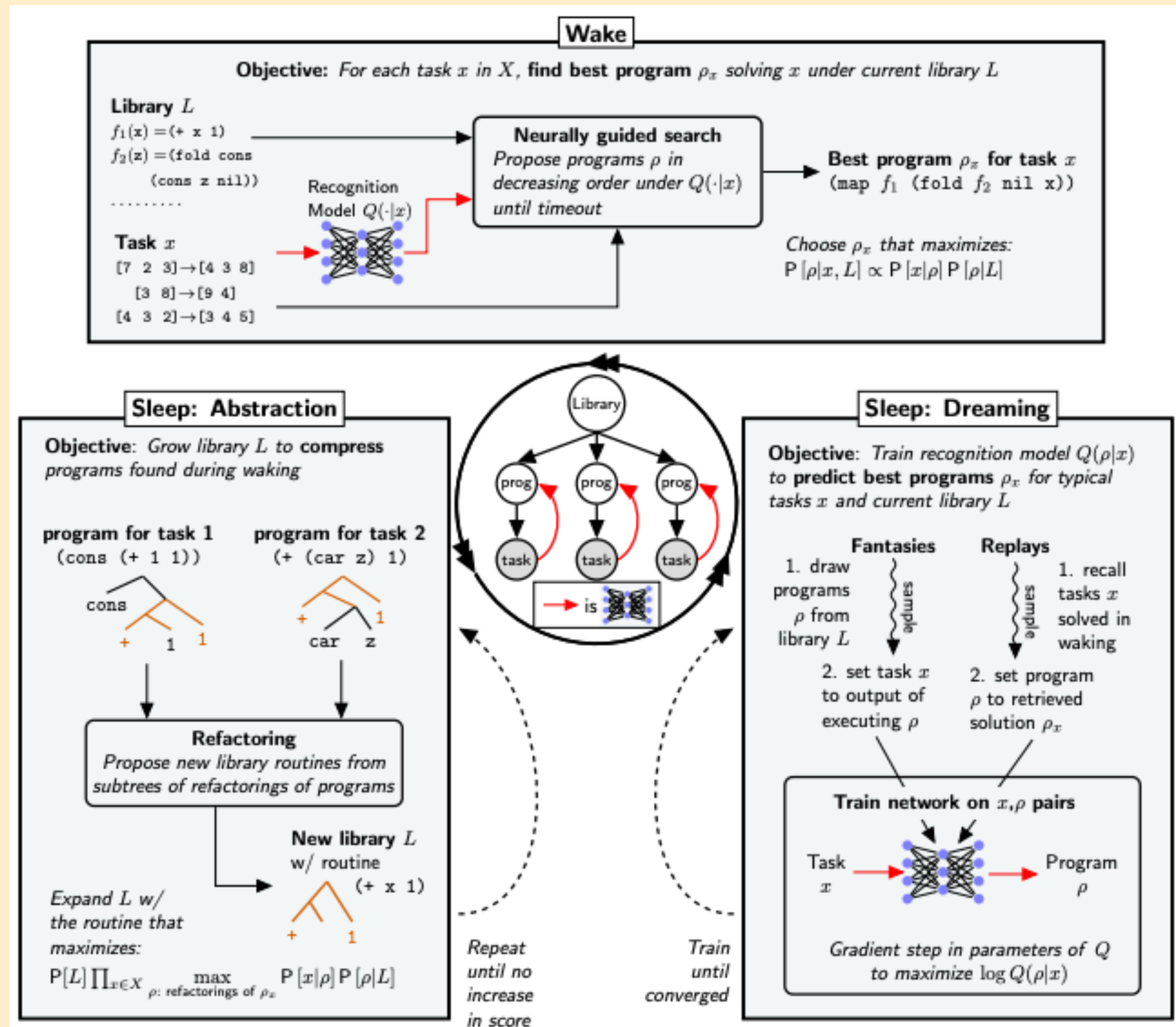


# Wake-Sleep Algorithm

- Inspired by Hinton et al., (1995)
- Wake**: find the best program to solve the current task using a recognition model (neural network)

$$\arg \max_{\pi} P(\mathcal{D} | \pi)$$

- Sleep**: Update  $P(\pi)$ 
  - Abstraction**: Grow library to find more compressible programs
  - Dreaming**: Train recognition model by sampling programs that solved previous experienced tasks (*replays*) and by sampling tasks that can be solved by programs in the current library (*fantasies*)



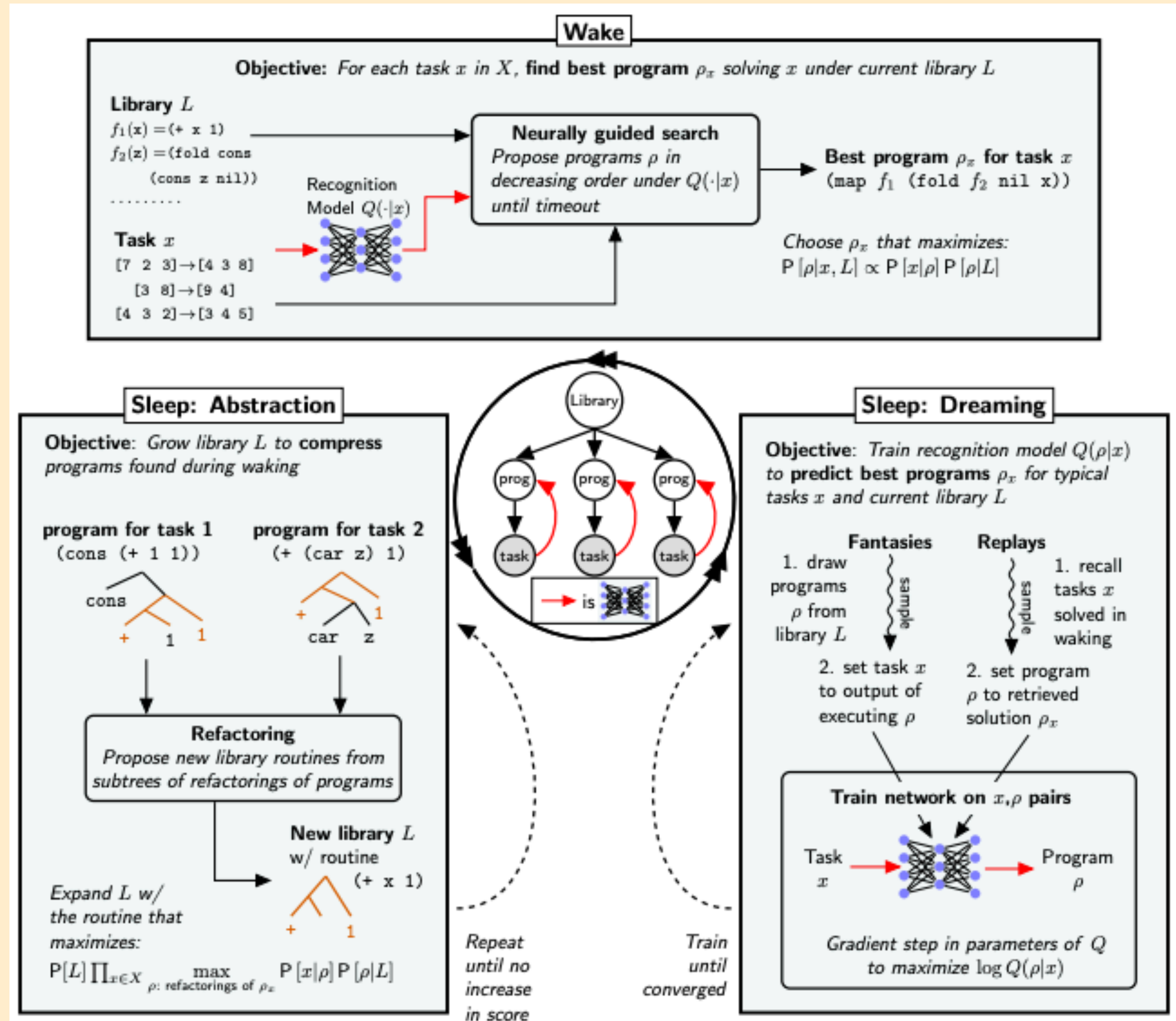


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# Current work in my lab

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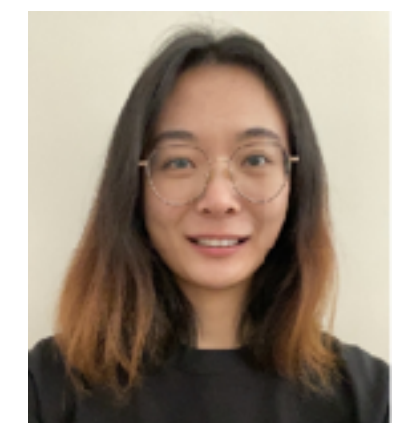
Hanqi Zhou



David Nagy

- Can program induction inform us about how people represent the world? (Fodor, 1975; Piantadosi et al., 2016; Dehaene et al., 2022)

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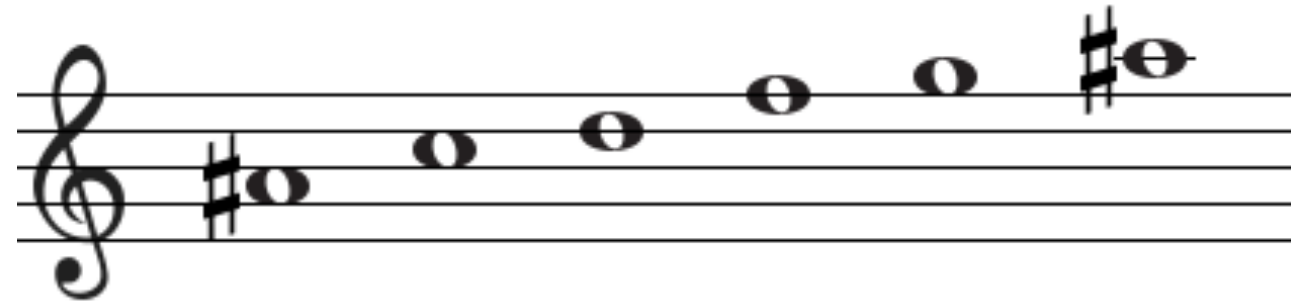


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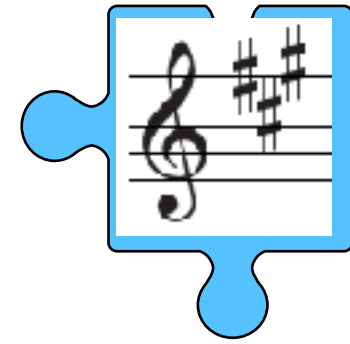
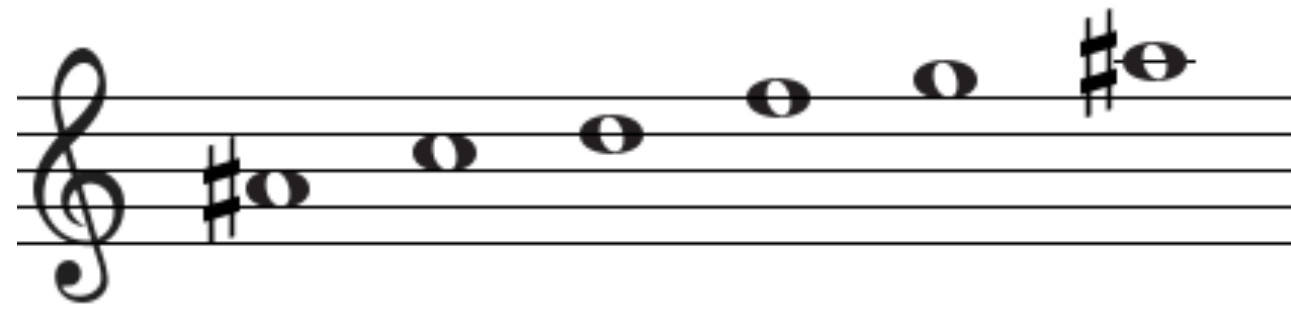


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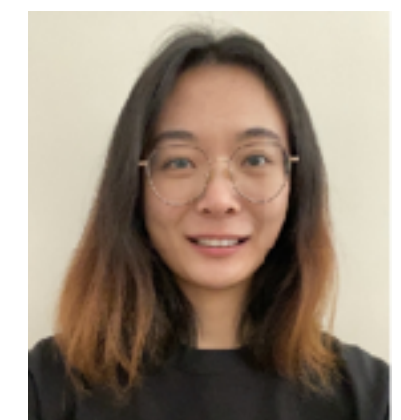


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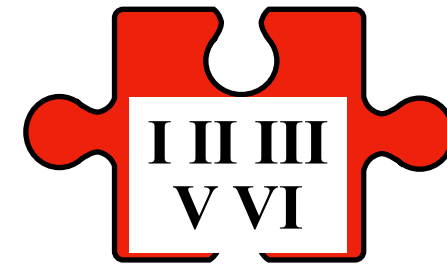
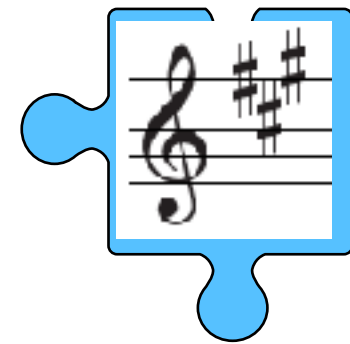
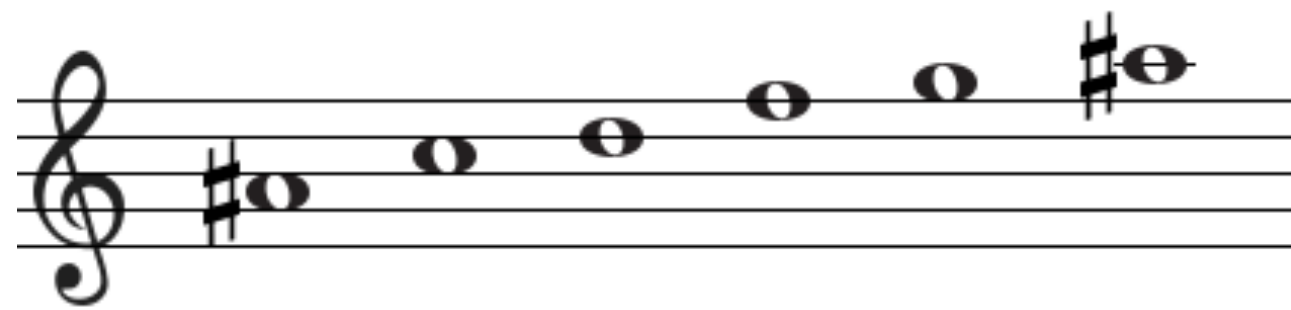


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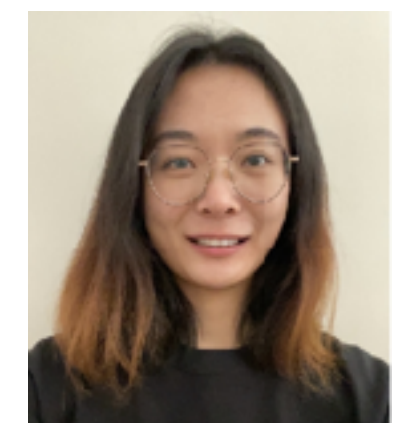


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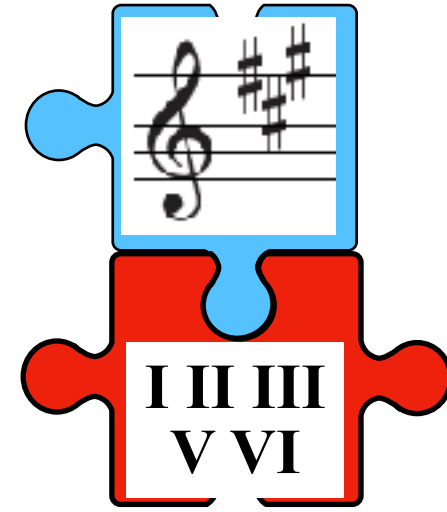
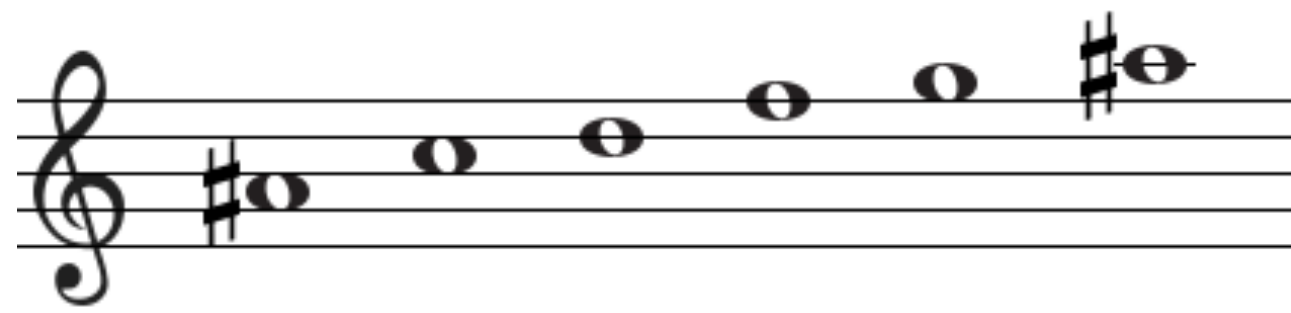


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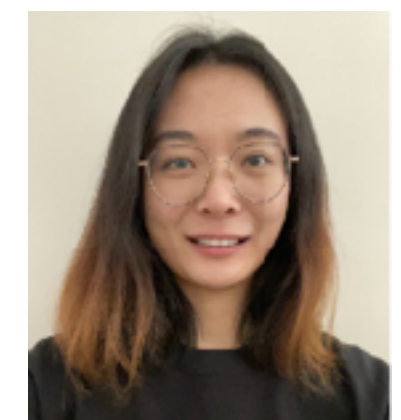
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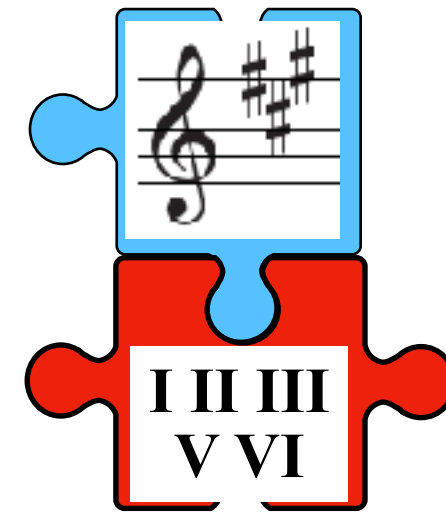
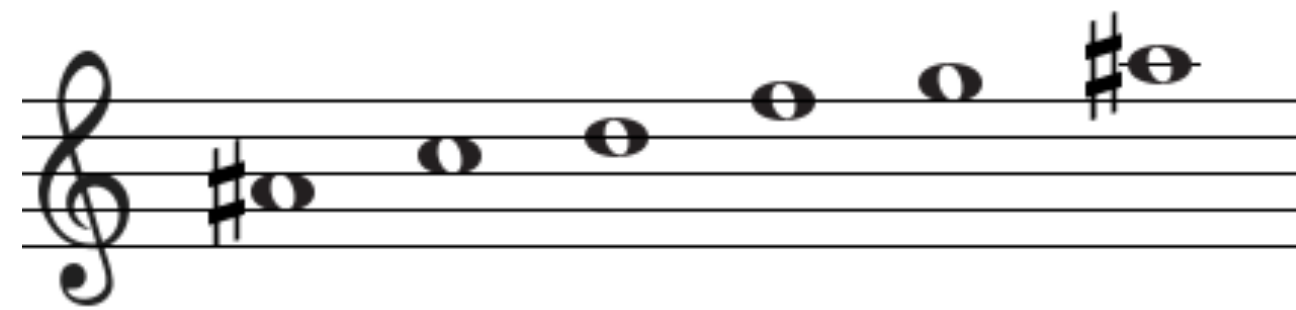


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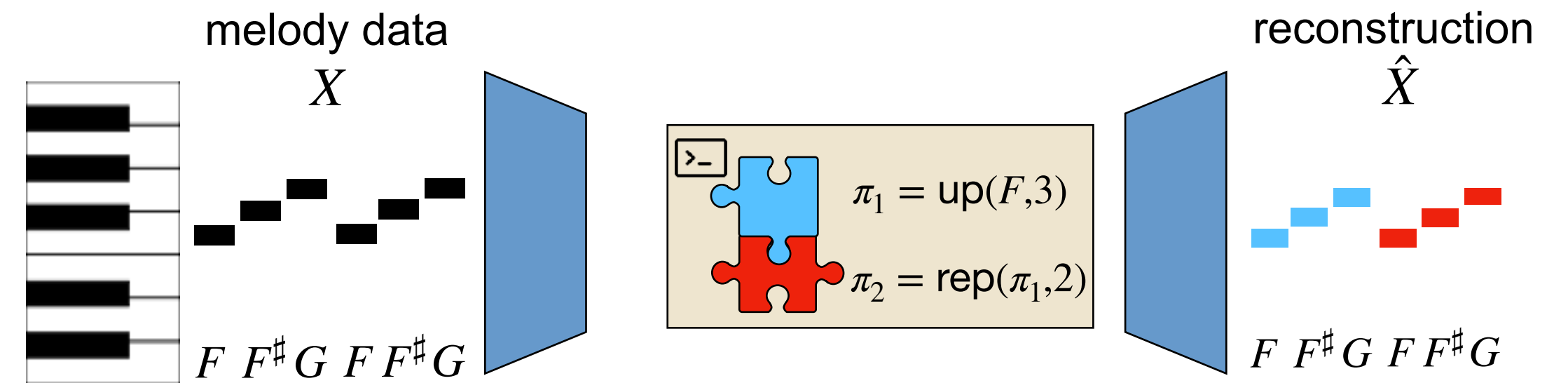


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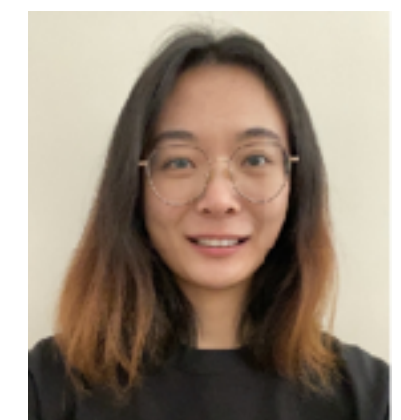
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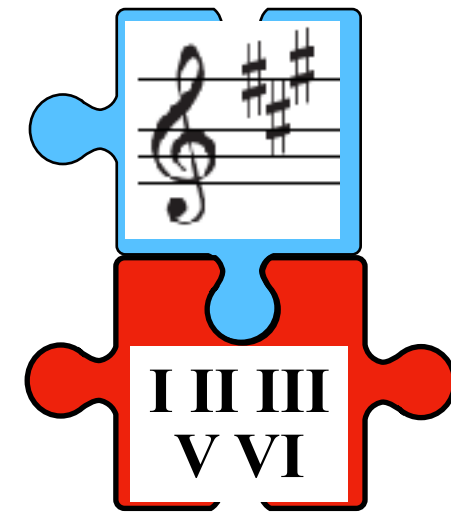
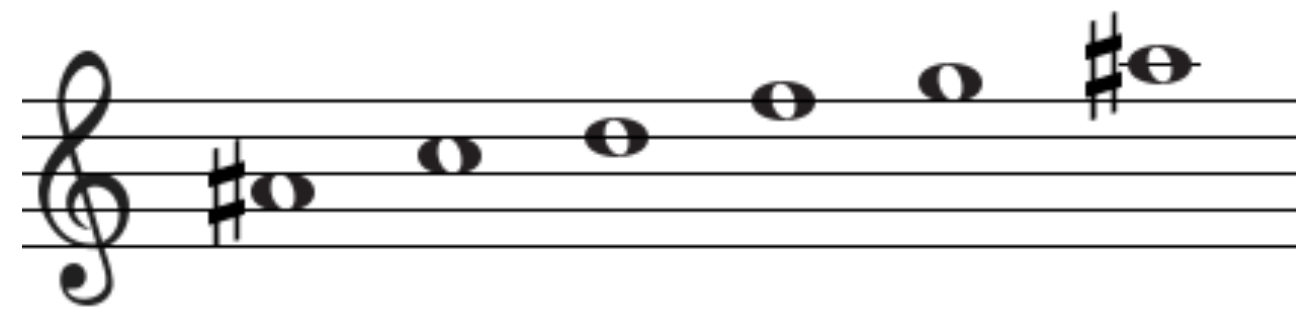


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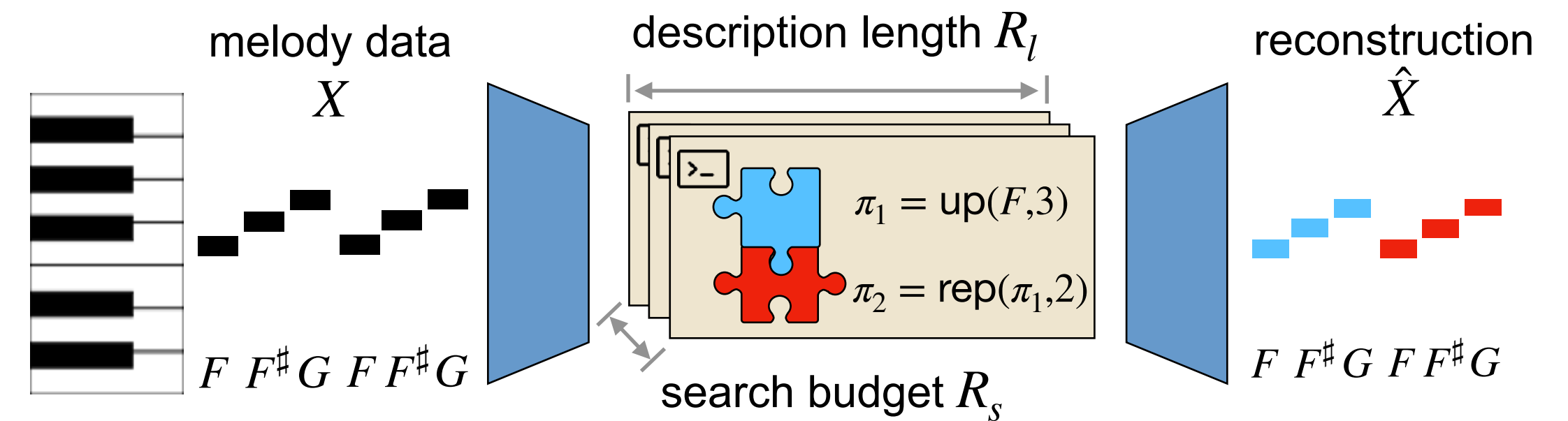
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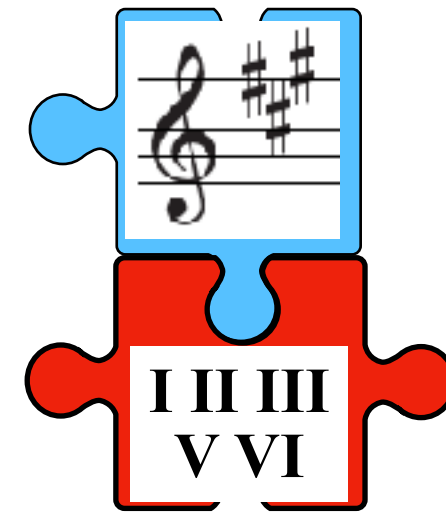
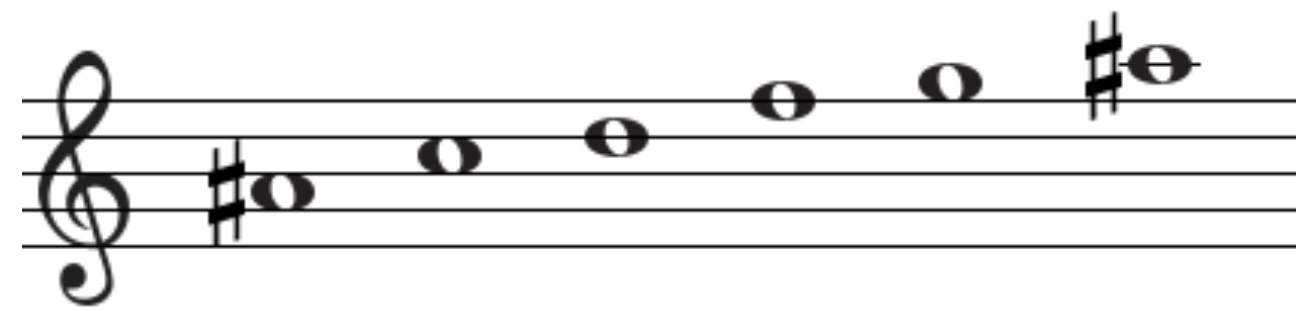


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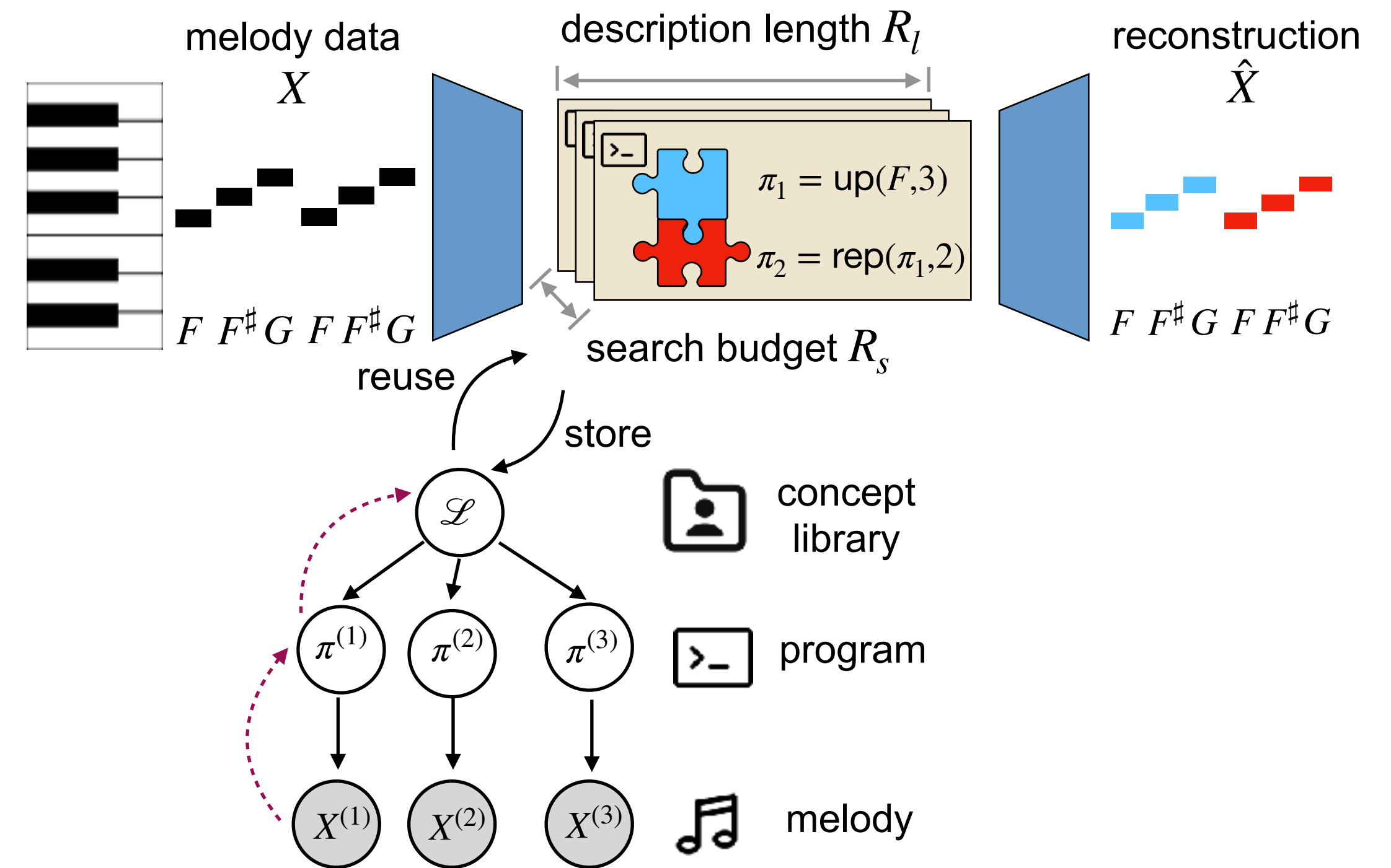
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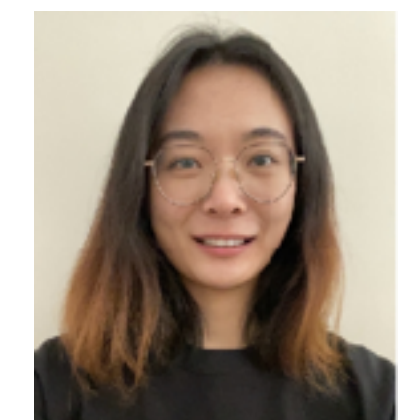
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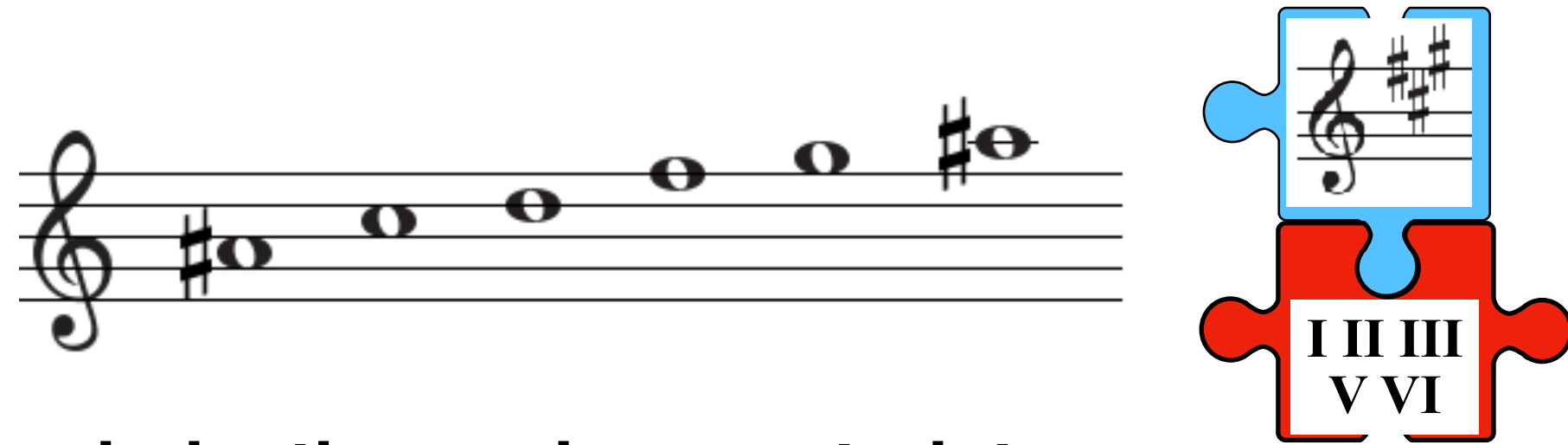


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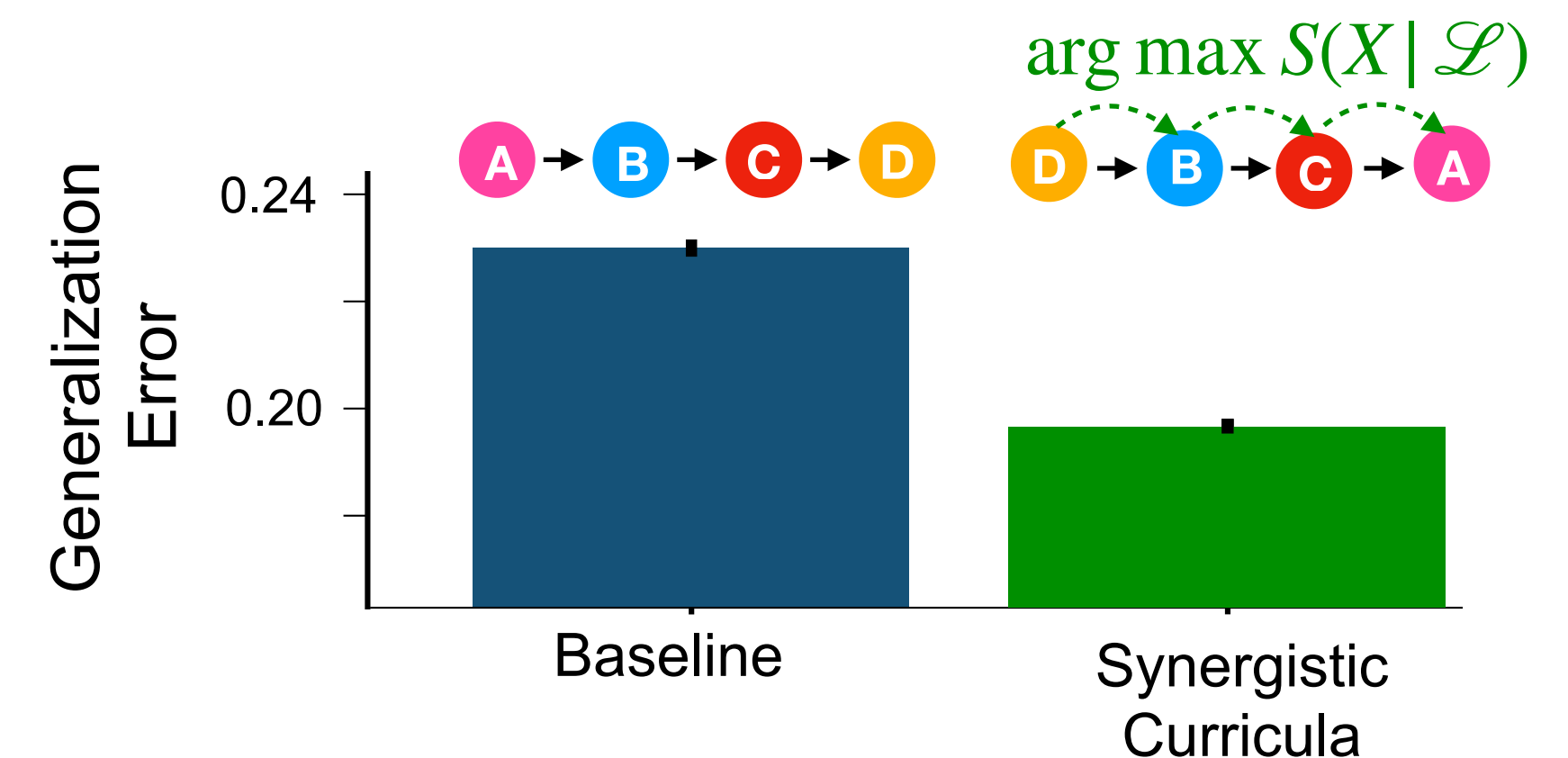
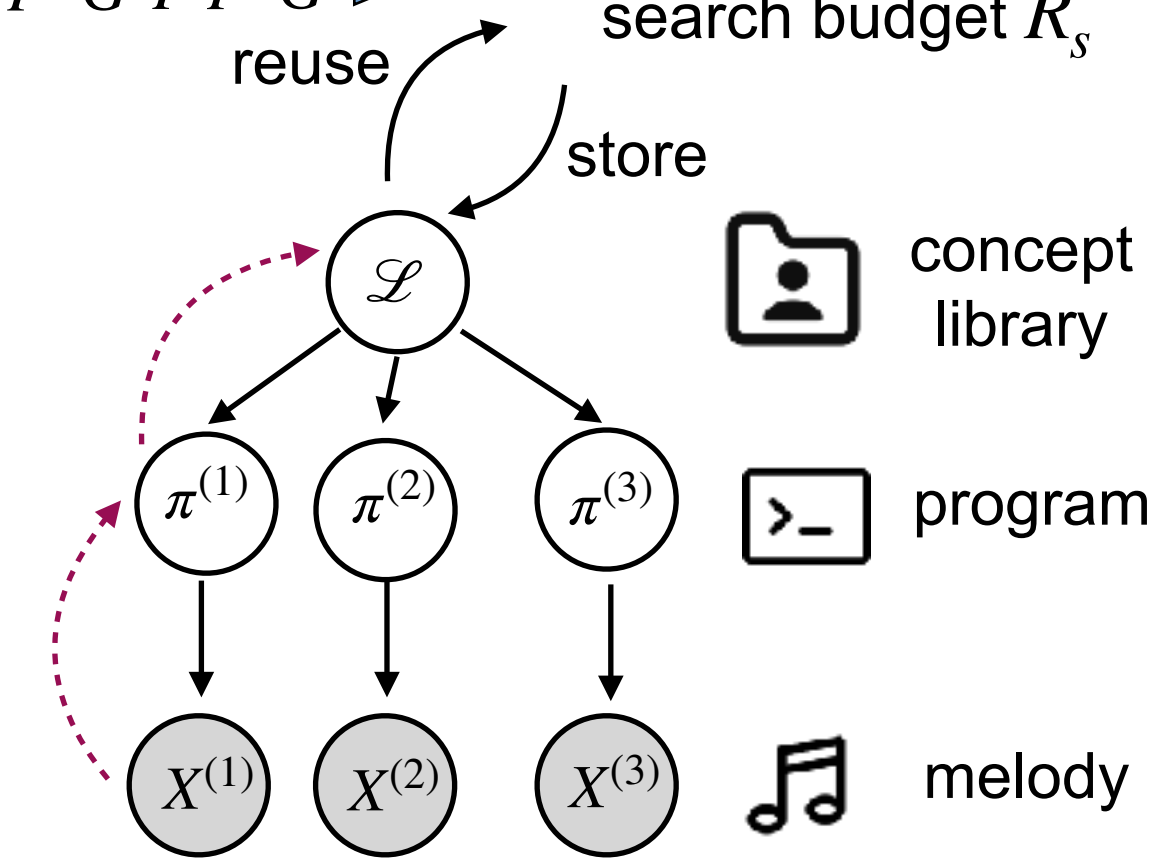
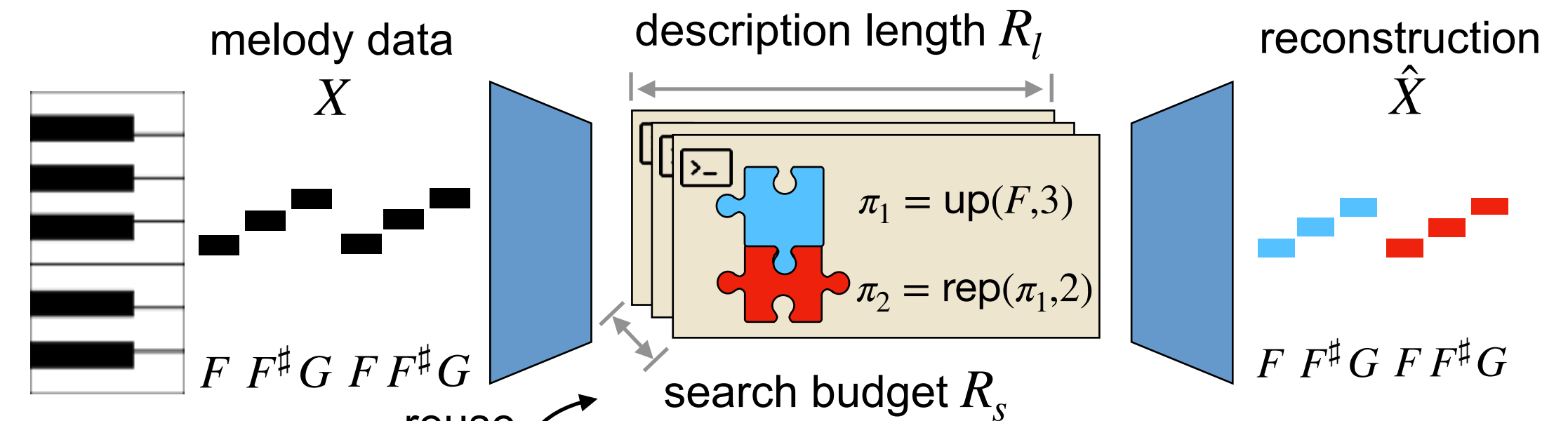
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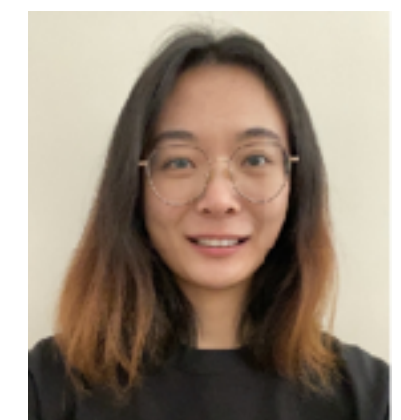


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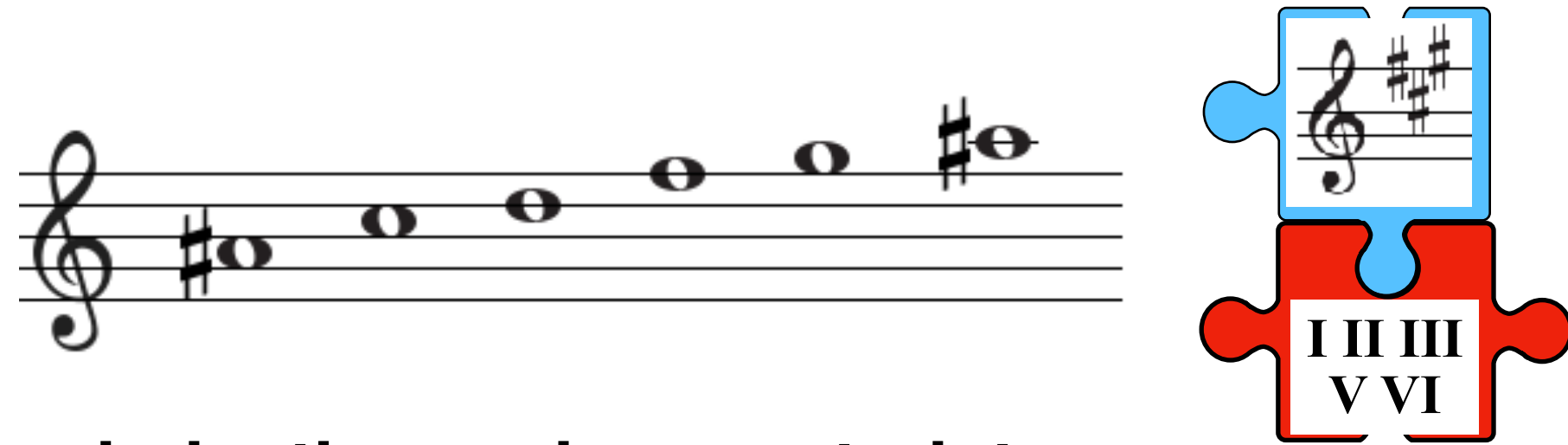


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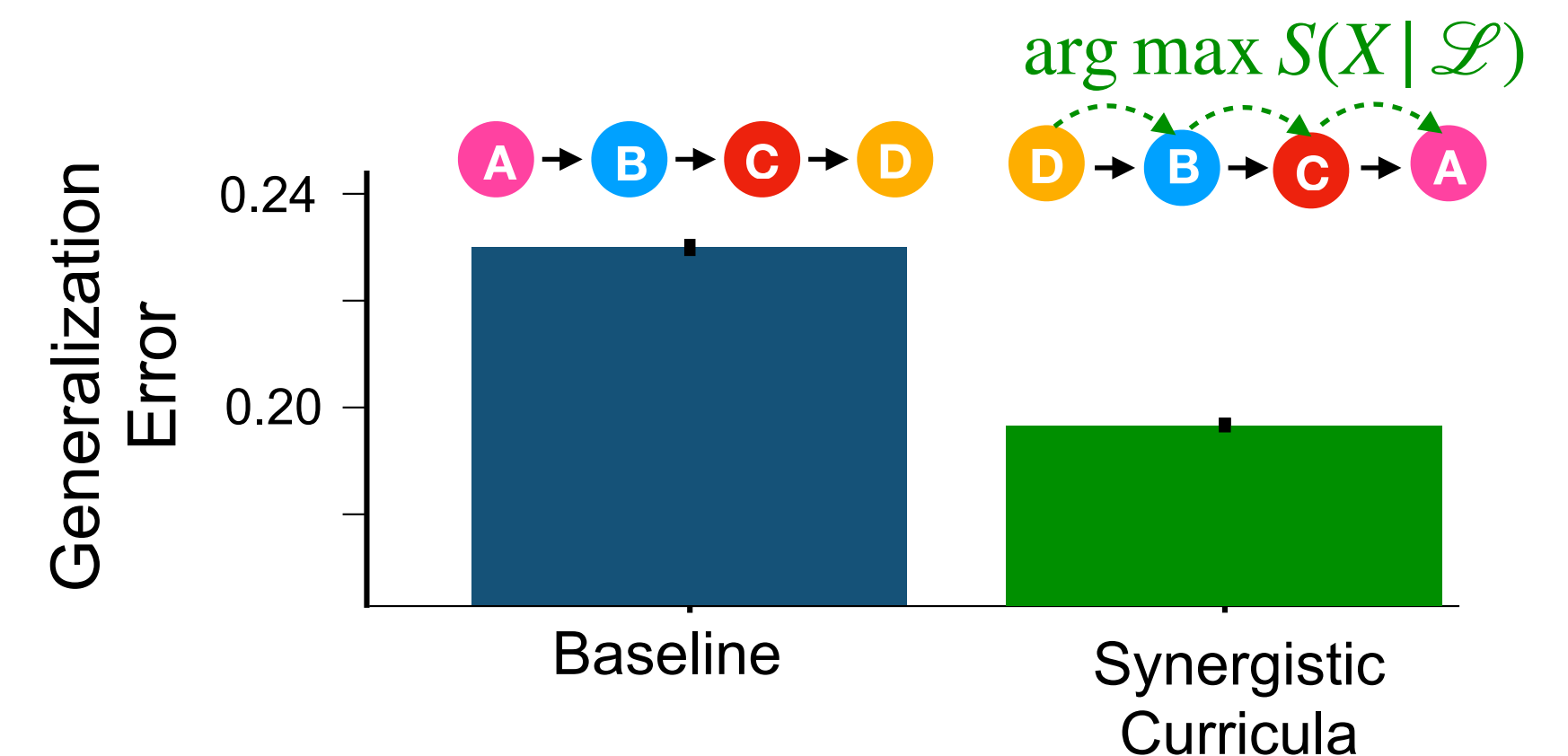
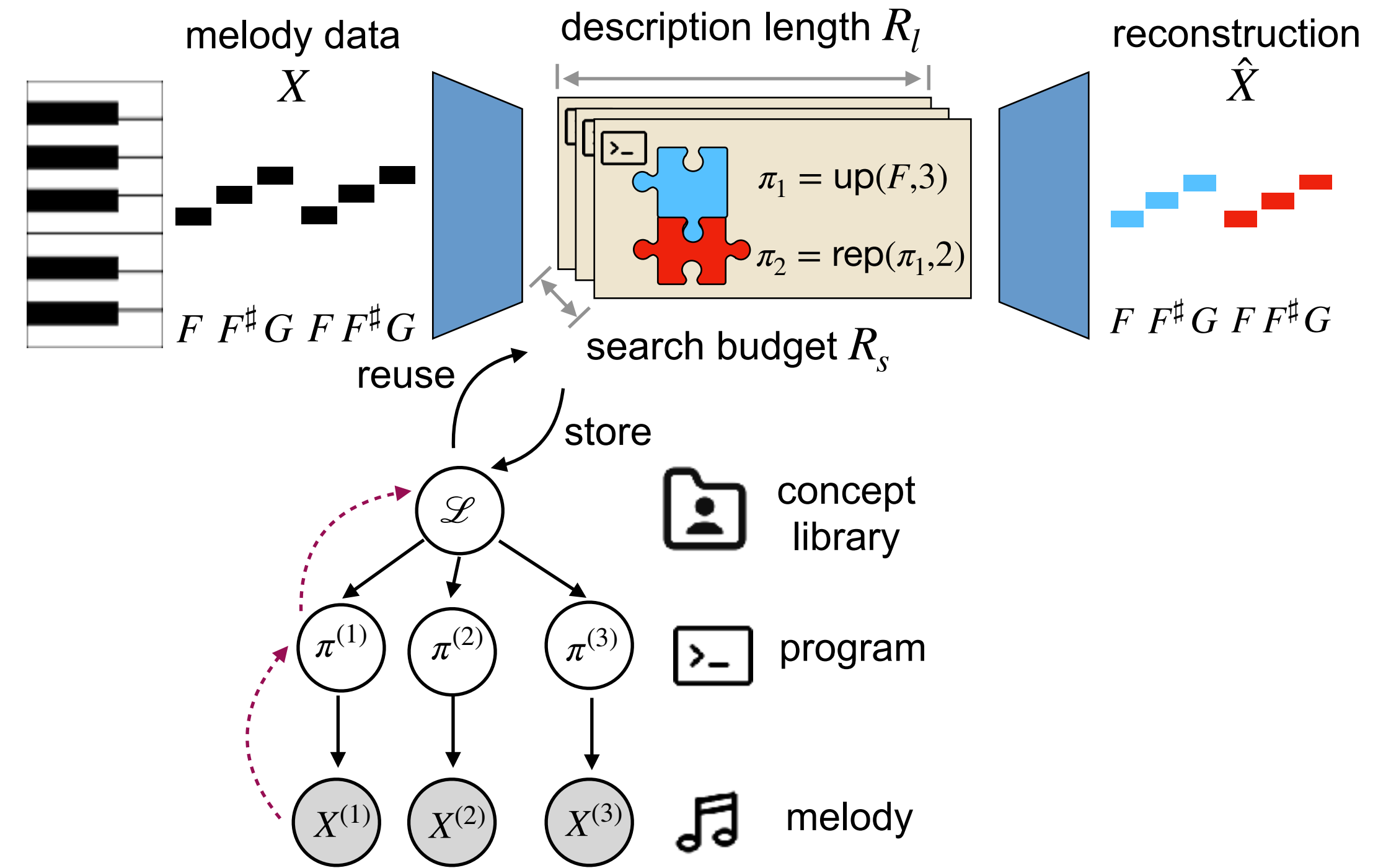


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- **Next steps**

- Test our predictions on human learners




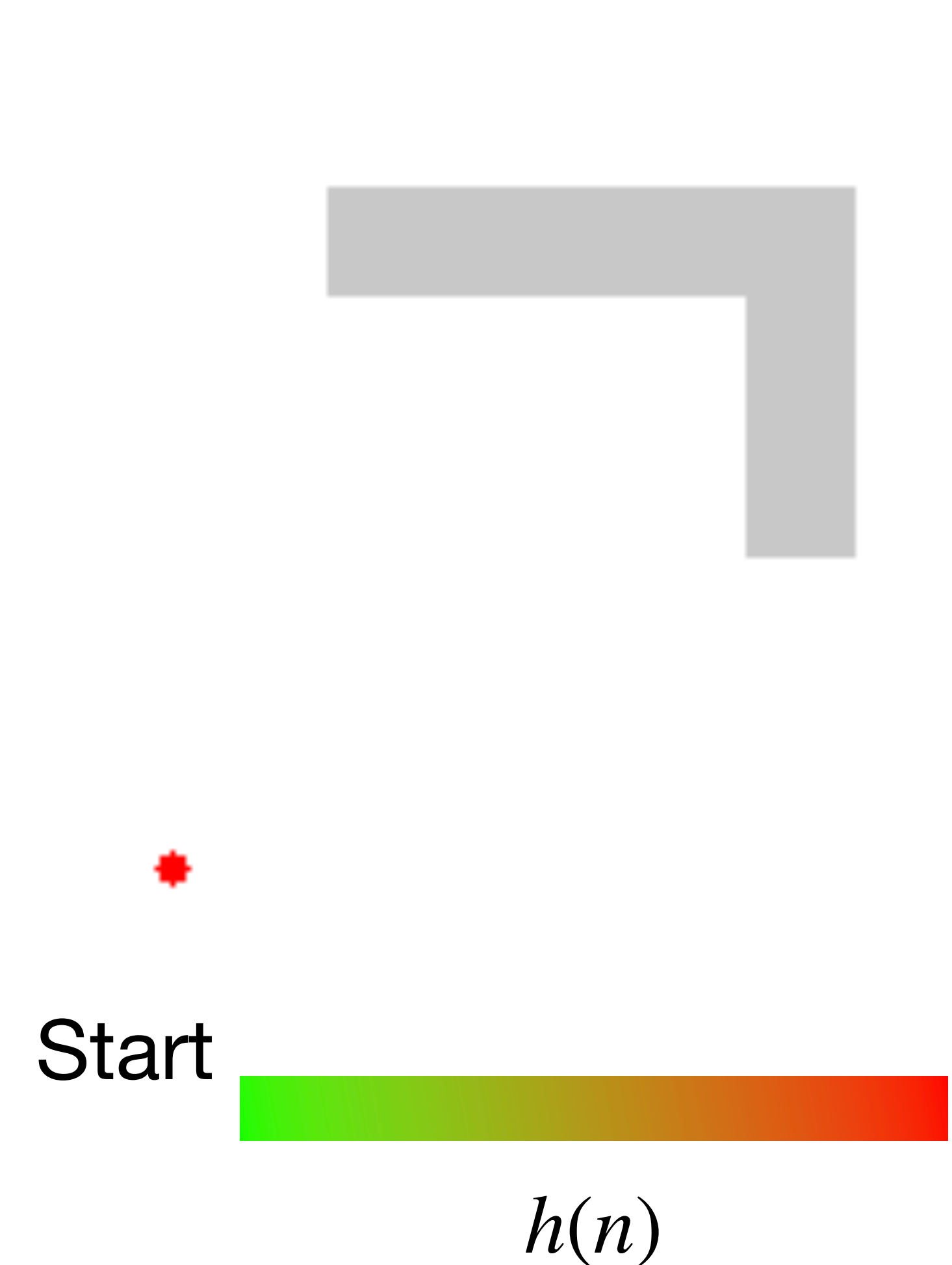
# Learning as Search

- A big part of what makes symbolic AI difficult is **search**
  - Representing relations between all possible symbols creates a combinatorial explosion
  - There are (typically) no gradients for symbolic representations
- Learning can thus be understood as a search problem
  - Finding which rules/programs capture data
  - Finding which hypotheses to test
- One of the major contributions of symbolic AI research was developing search algorithms
  - $A^*$
  - Montecarlo Tree Search




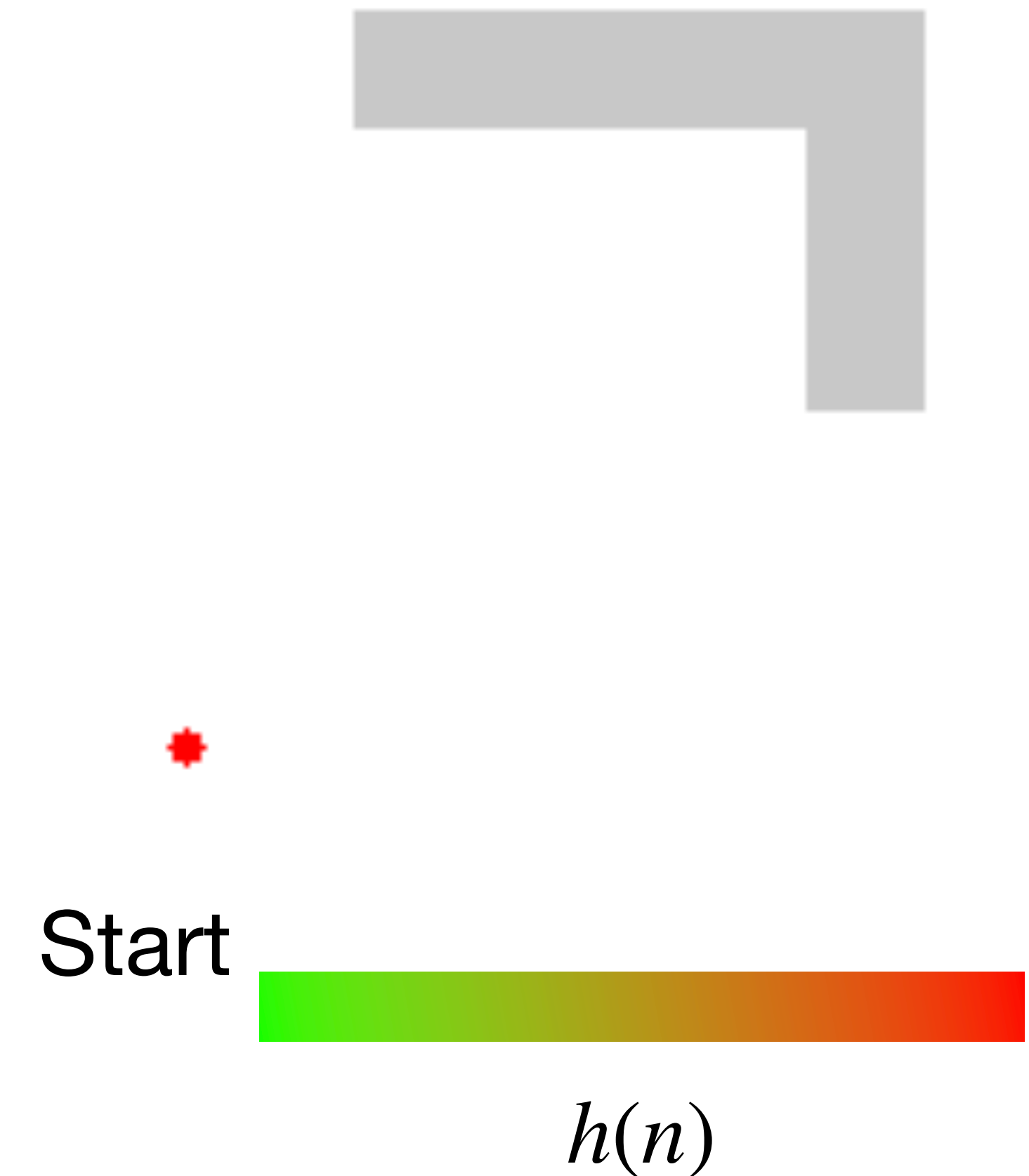
# A\* Heuristic Search

- One of the most popular methods for path-finding and search over graphs (Hart et al., 1968)
- Expand the path by choosing candidate node  $n$   that minimizes cost function  $f(n) = g(n) + h(n)$ 
  - *Keep the current path short*:  $g(n)$  is the cost of the path so far from the start to  $n$ 
    - Costs can also represent complexity (i.e., the number of symbolic operations)
  - *Move towards the goal*:  $h(n)$  is a **heuristic** that estimates the cost of the cheapest remaining path from  $n$  to the goal (often Euclidean distance)
    - The heuristic avoids calculating the actual remaining cost to the goal, which is very costly
- More efficient than **backwards induction**, but intractable for any interesting program induction problems




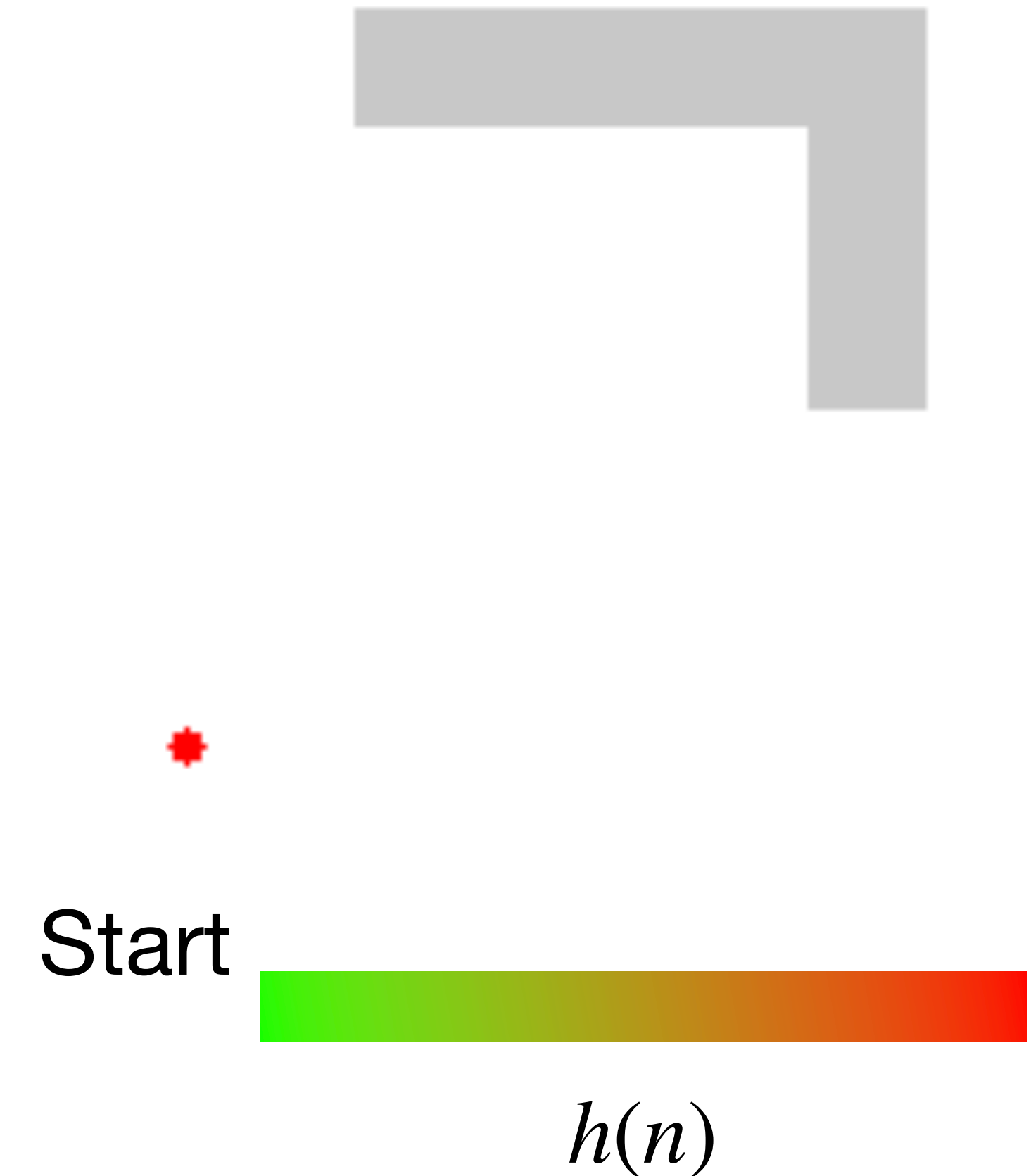
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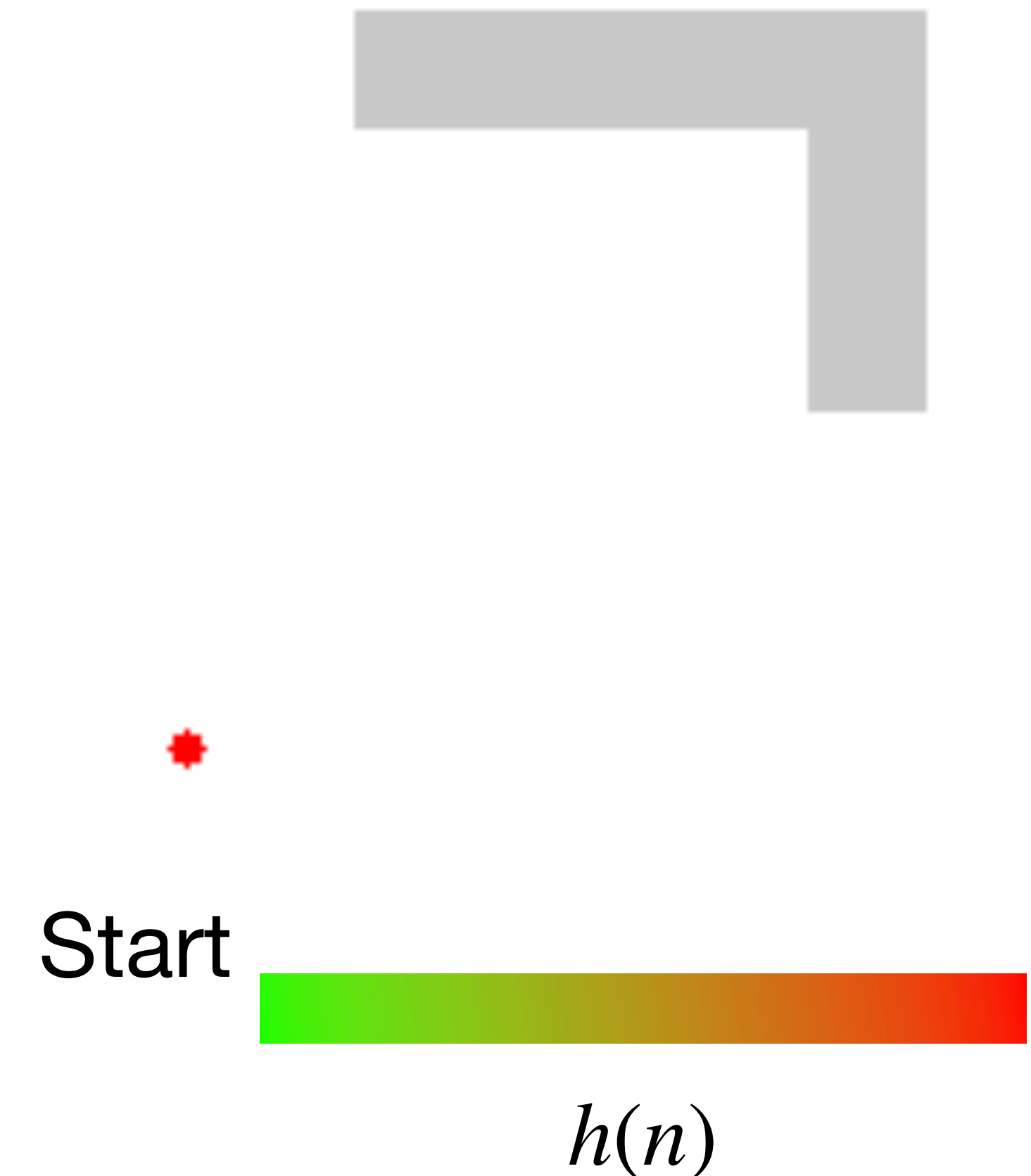


**Heuristic:** a problem-solving strategy or method that is not guaranteed to find the optimal solution, but is designed to find a satisfactory solution in a reasonable amount of time



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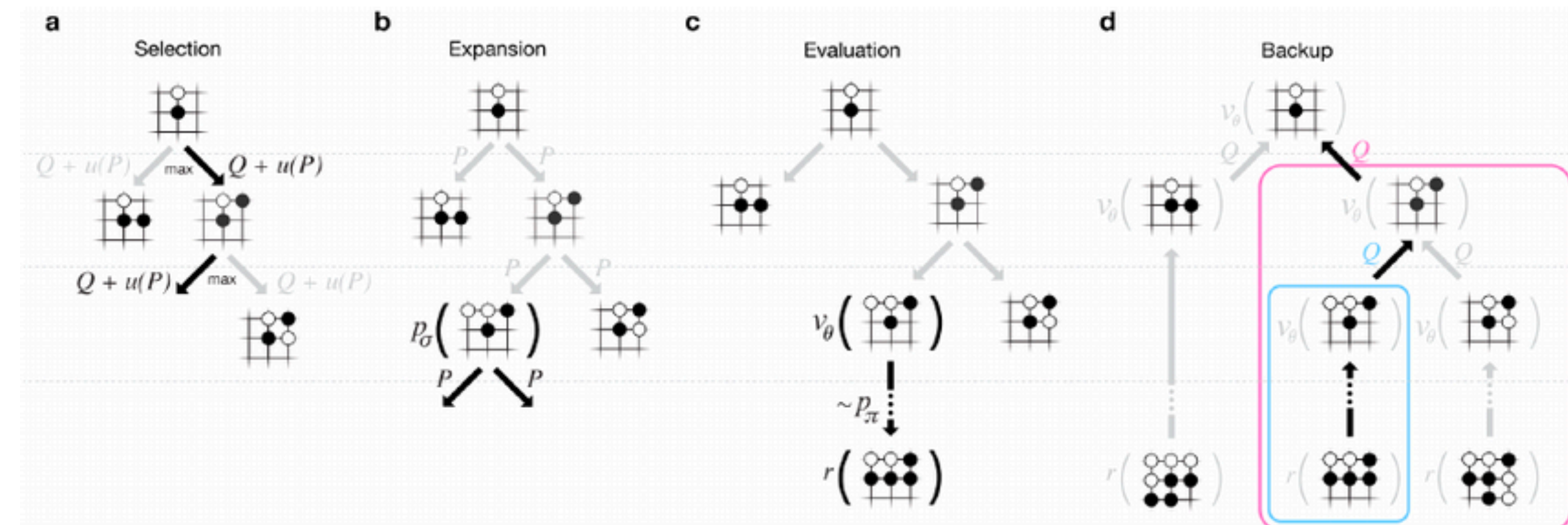
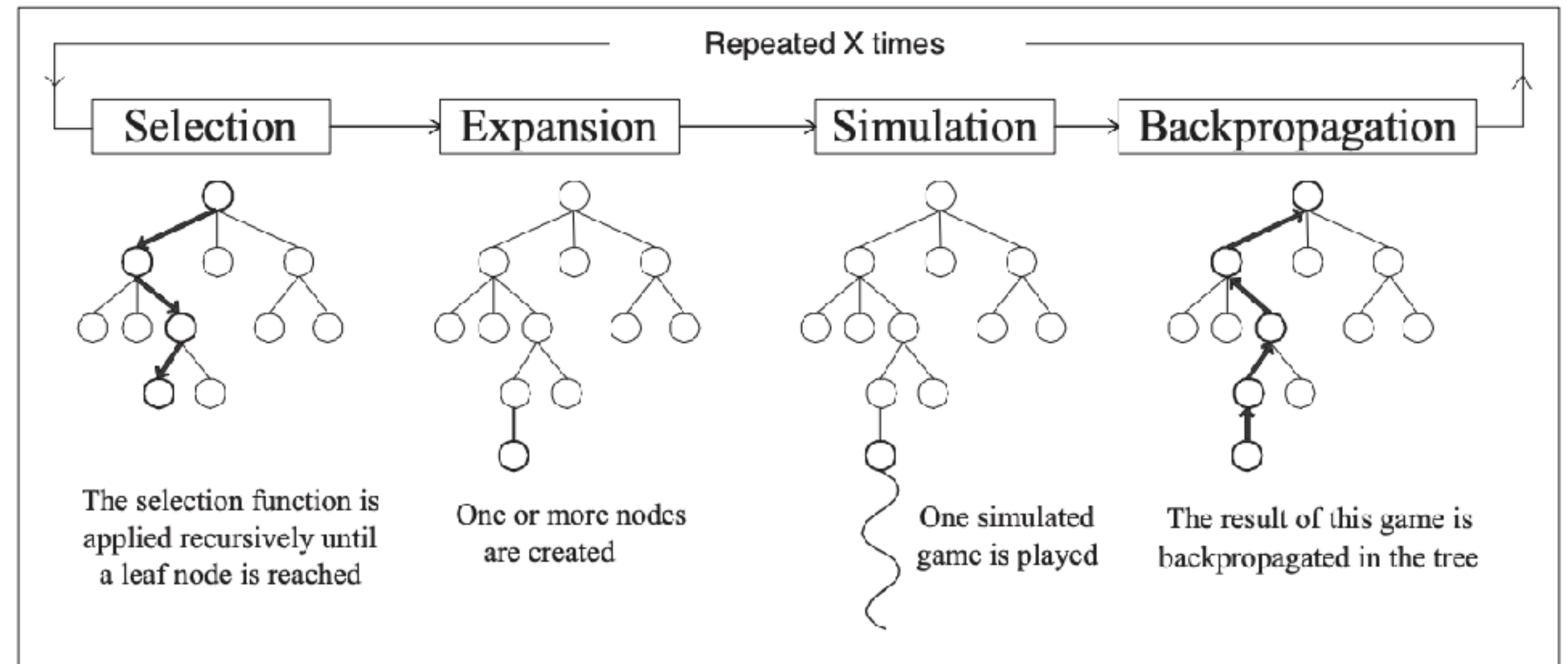


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**Backwards induction:** determining a sequence of optimal choices by reasoning from the endpoint of a problem back to the beginning

# Monte Carlo Tree Search

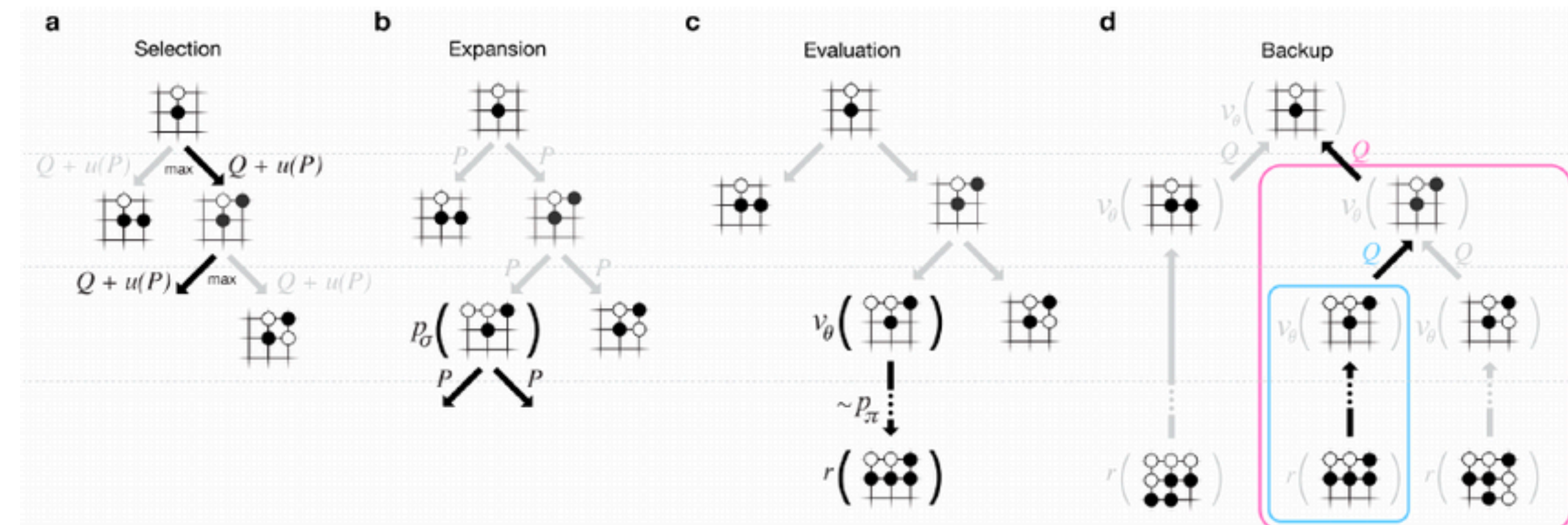
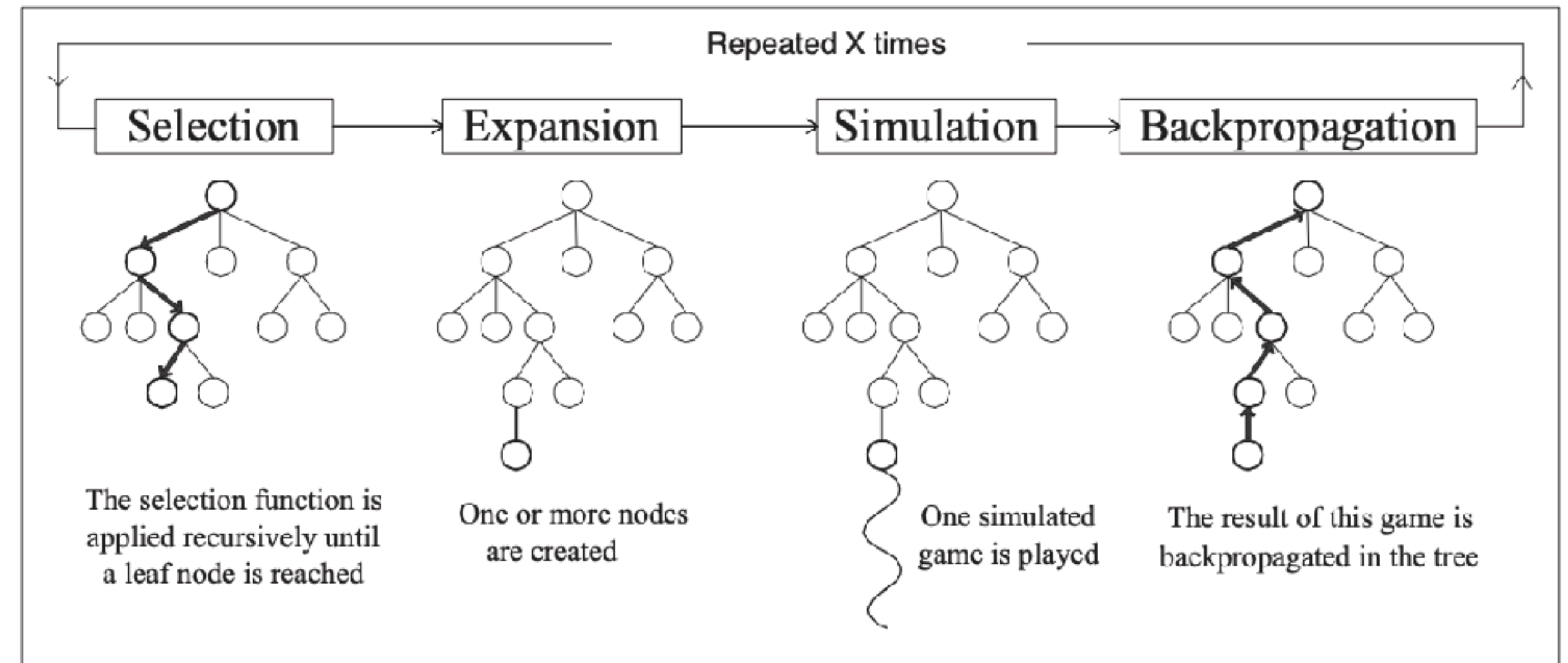
- A key mechanism in AlphaGo (Silver et al., 2016) and other modern RL algorithms
- **Select** nodes for expansion (often using a heuristic based on reward + *information gain*)
- **Expand** node and perform **simulations**
- **Backpropogate** the value of the child to the parent node
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**Information gain:** The amount of information gained by an observation (i.e., expanding a node). Often approximated using count-based methods:

$$\uparrow \text{info gain} \propto \downarrow \text{fewer visits}$$



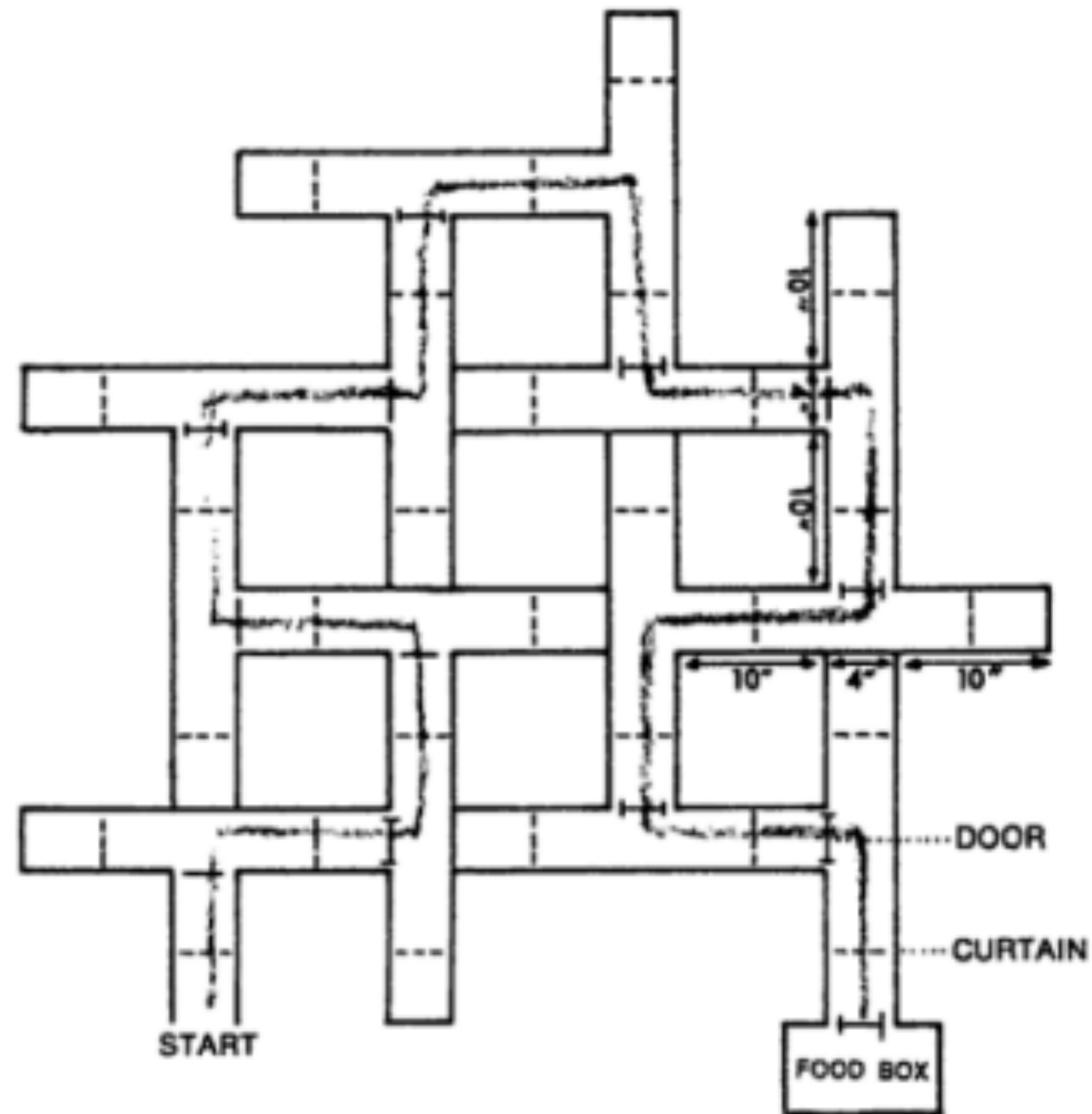
# Symbolic AI: Summary

- **Symbols** and relational **rules** are a powerful tool for describing the world
  - Capture rapid generalization and allow for compositional construction of new representations
  - Explicit formulation of relationships in the world that mirror our own Language of Thought and provides interpretable predictions
- **Learning is difficult** and rules can sometimes be too rigid
  - Compositional hypothesis space leads to a combinatorial explosion of possible symbolic representations, where search can be very costly
  - Learning is often framed as a search problem, where heuristic solutions provide a valuable aid
- **Neurosymbolic AI** might offer the best of both worlds by combining the fast learning of subsymbolic AI (i.e., neural networks) with the powerful abstractions of symbolic AI

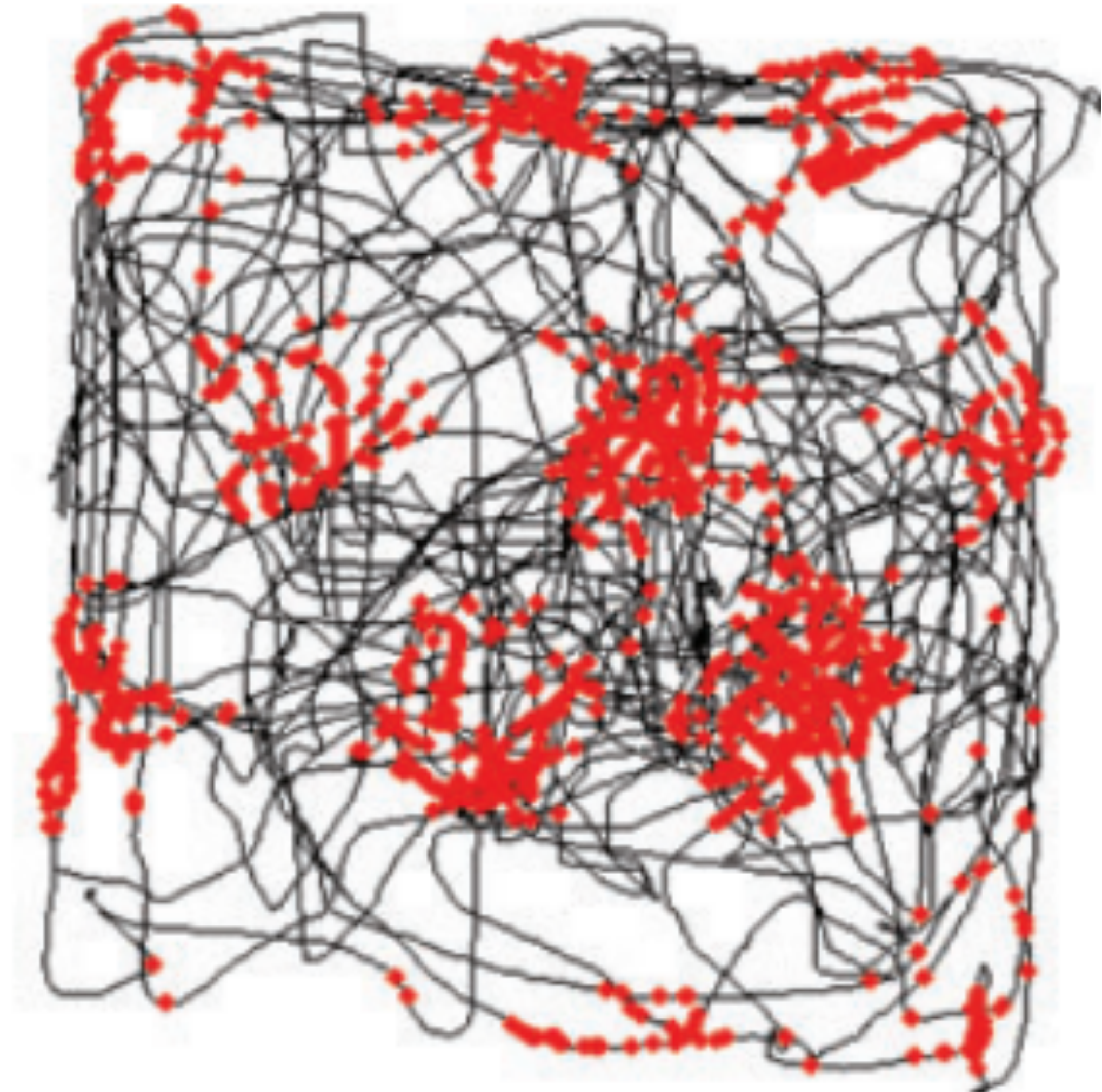
5 minute break



# Cognitive Maps



Tolman (1948)

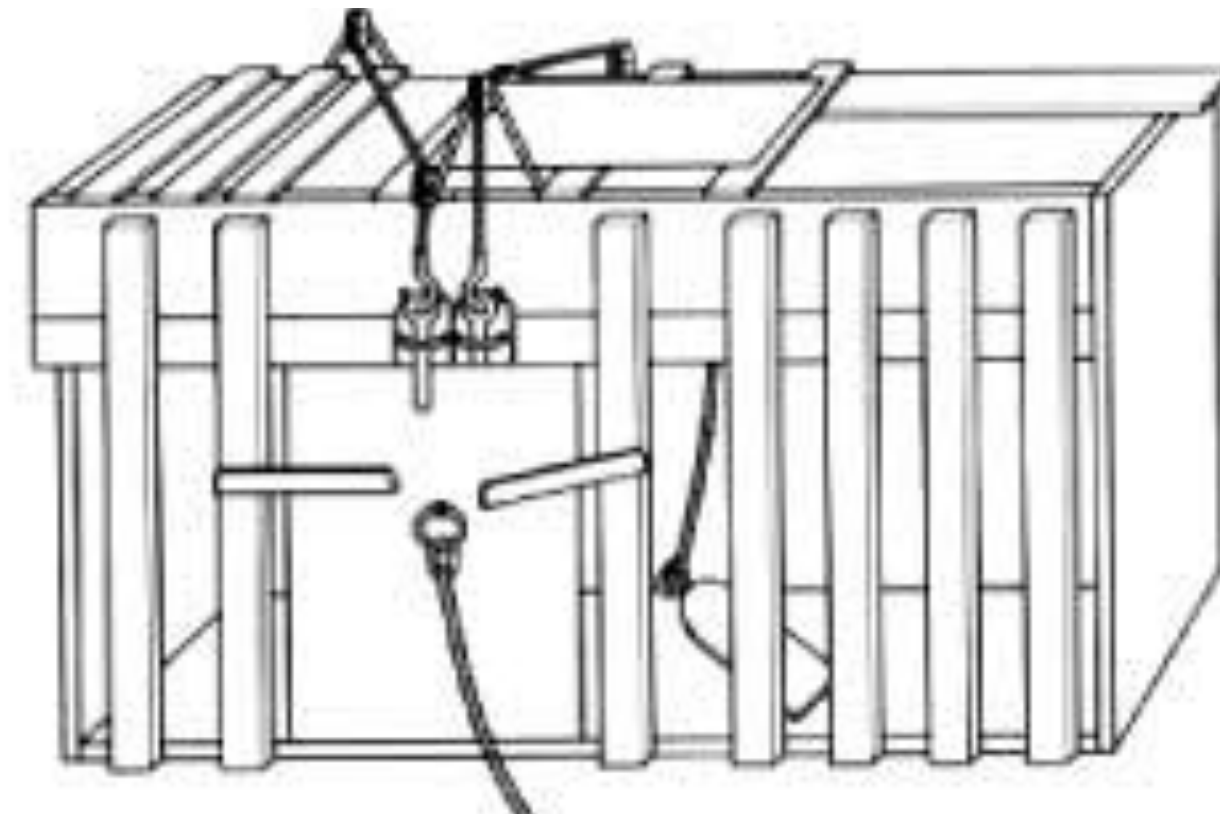


Moser et al., (2008)



The story so far ...

# Thorndike's (1911) Law of Effect

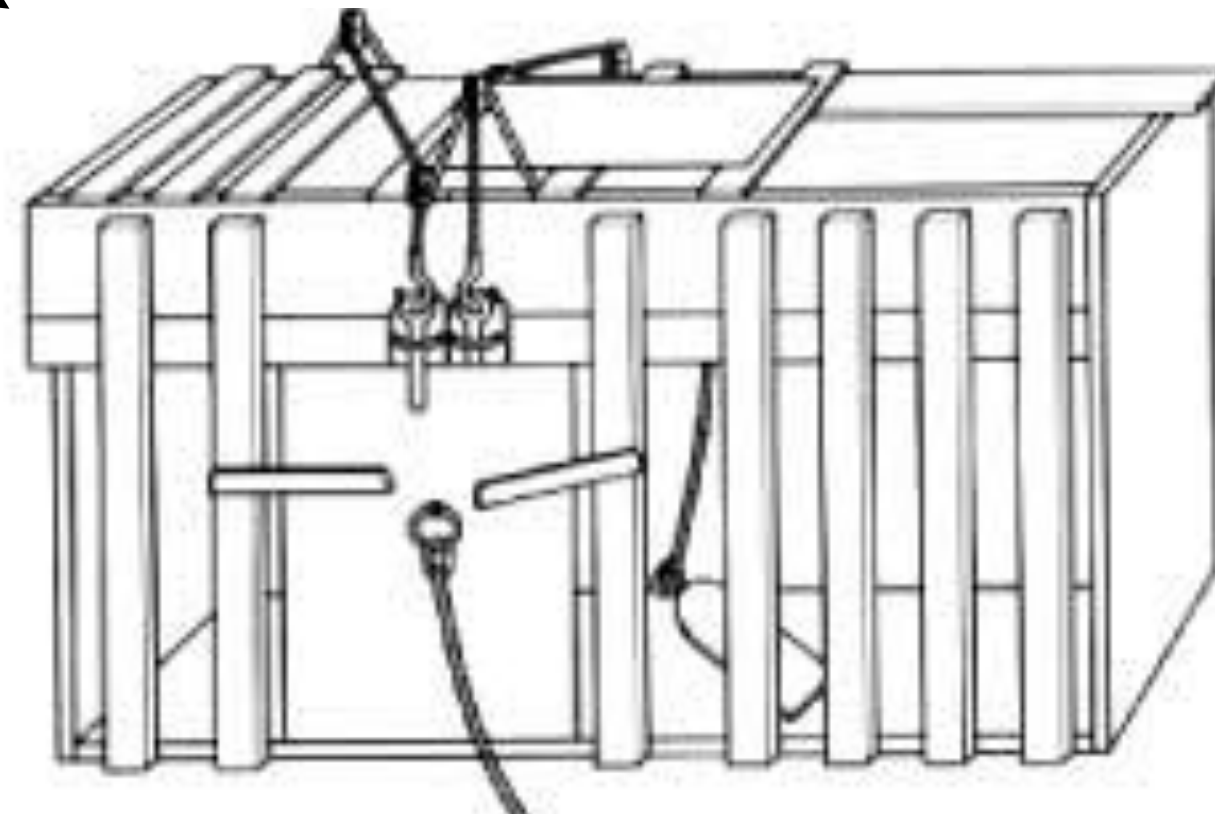


Puzzle Box

# Thorndike's (1911) Law of Effect



Cat



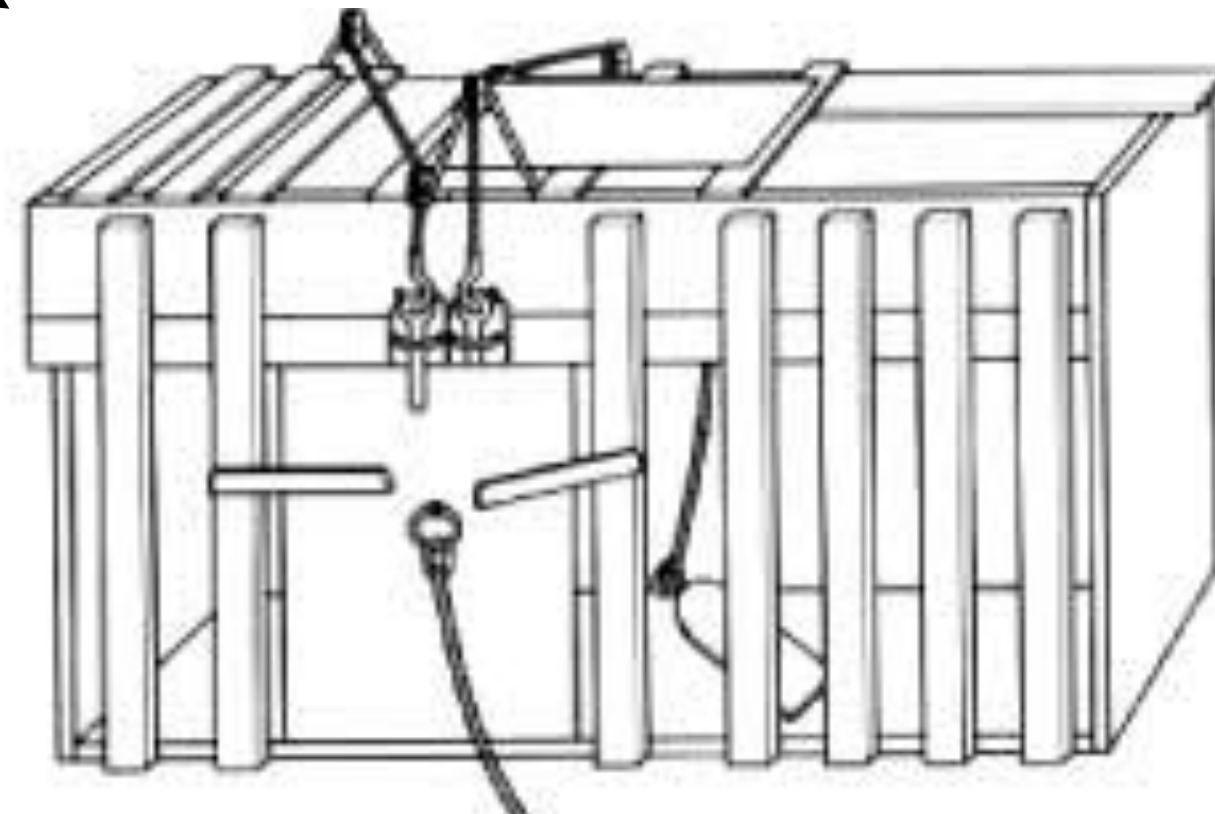
Puzzle Box



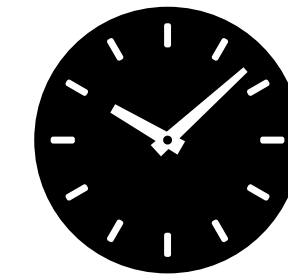
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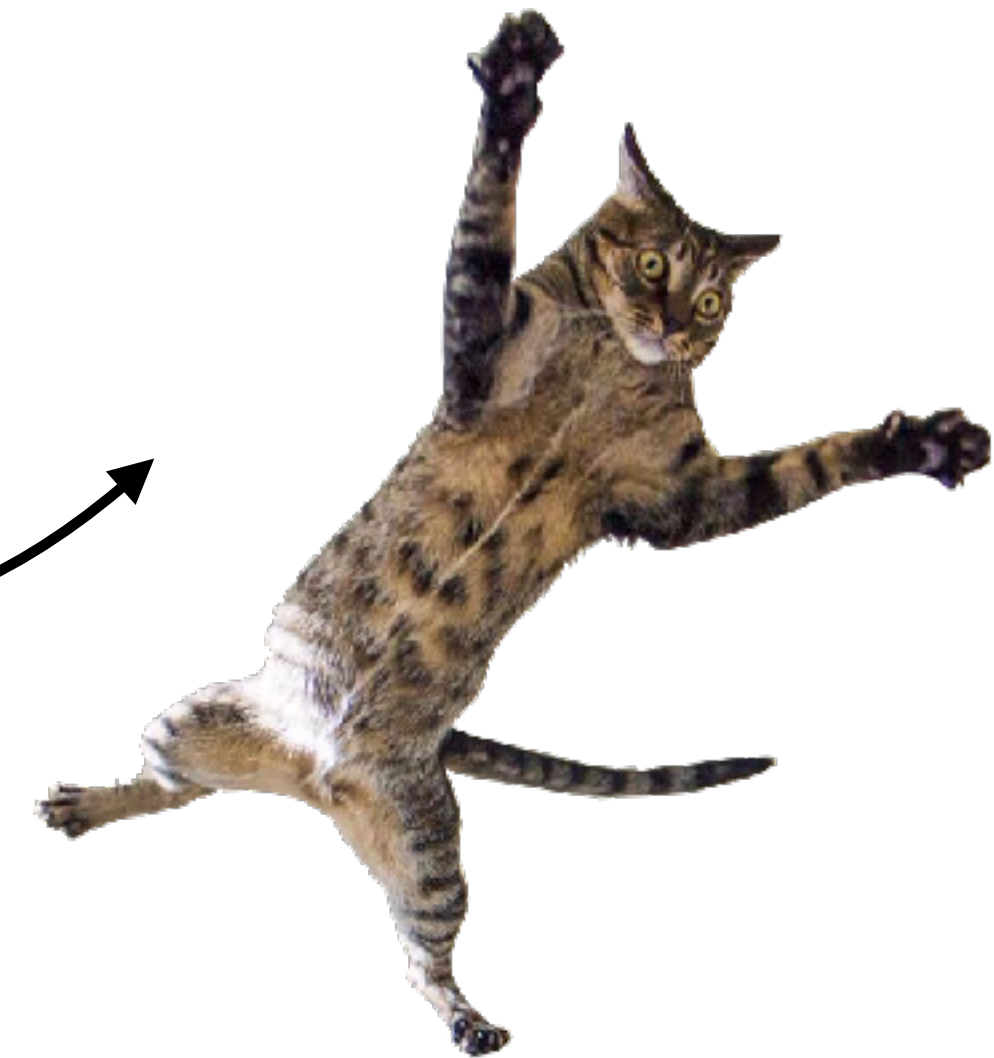
Cat



Puzzle Box



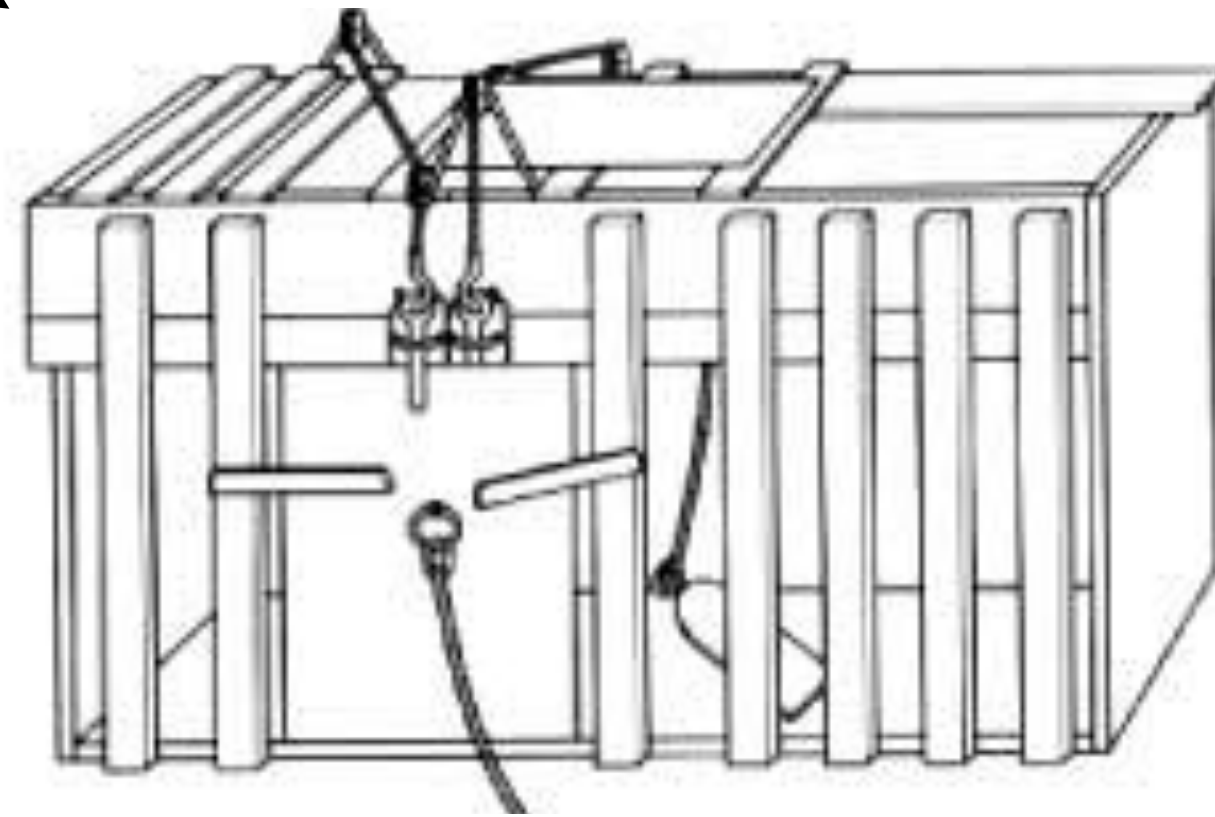
Time to escape



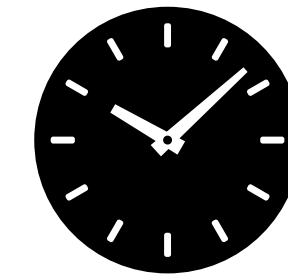
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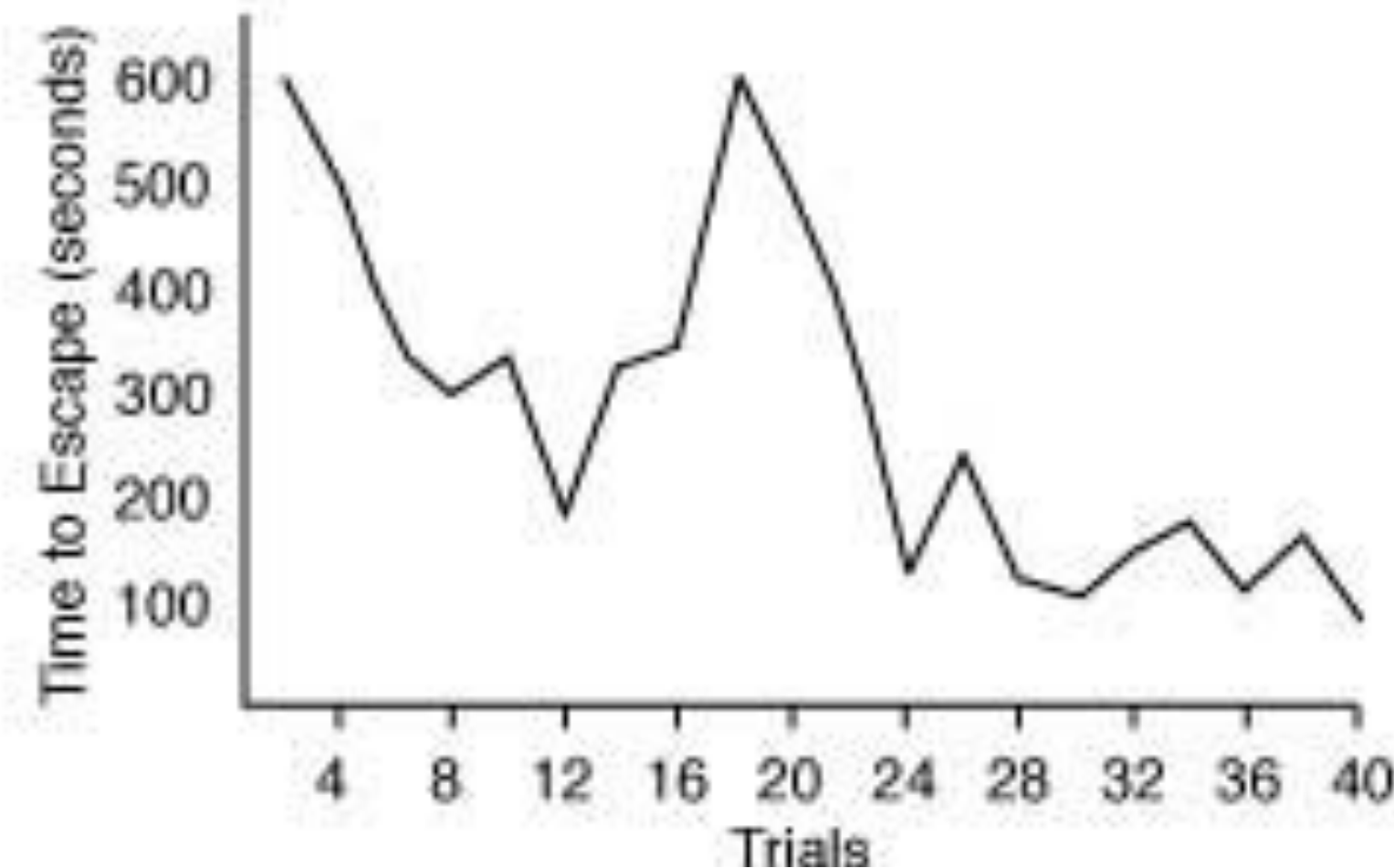
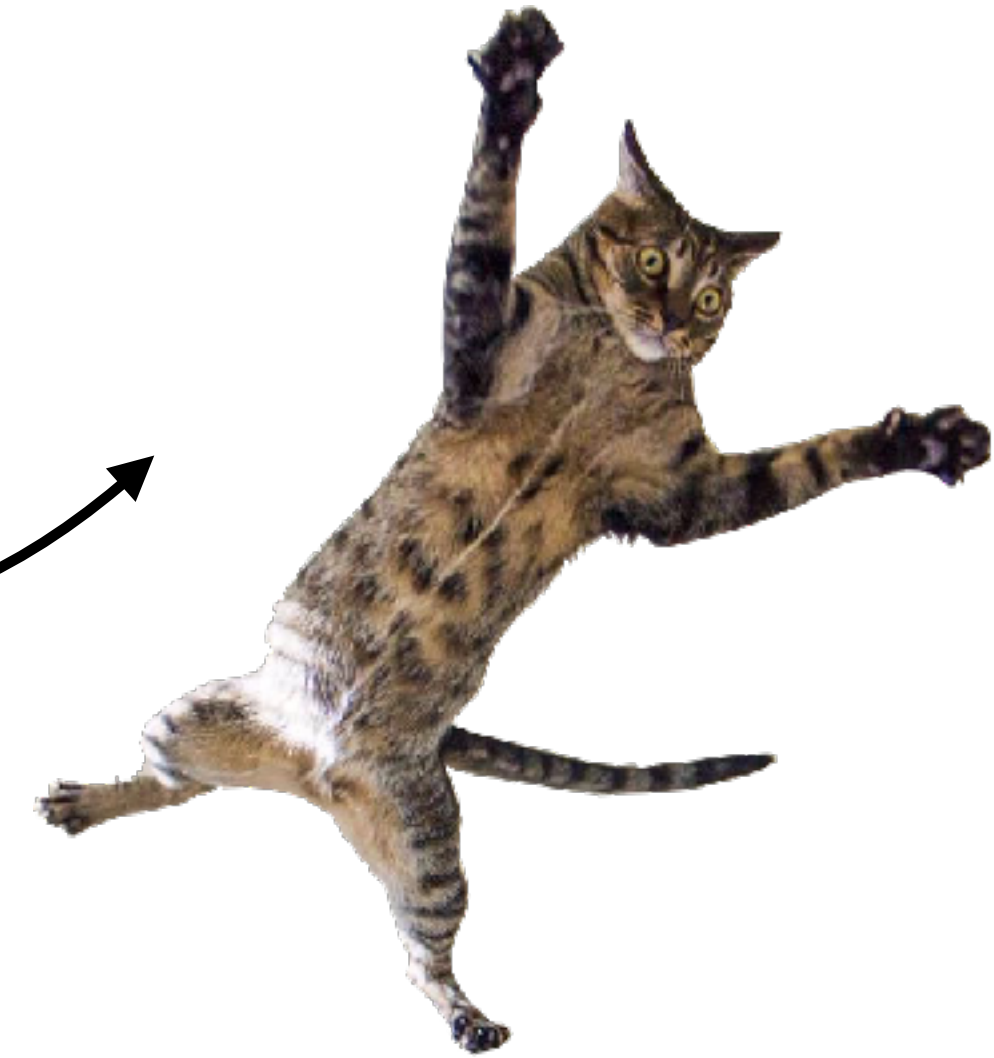
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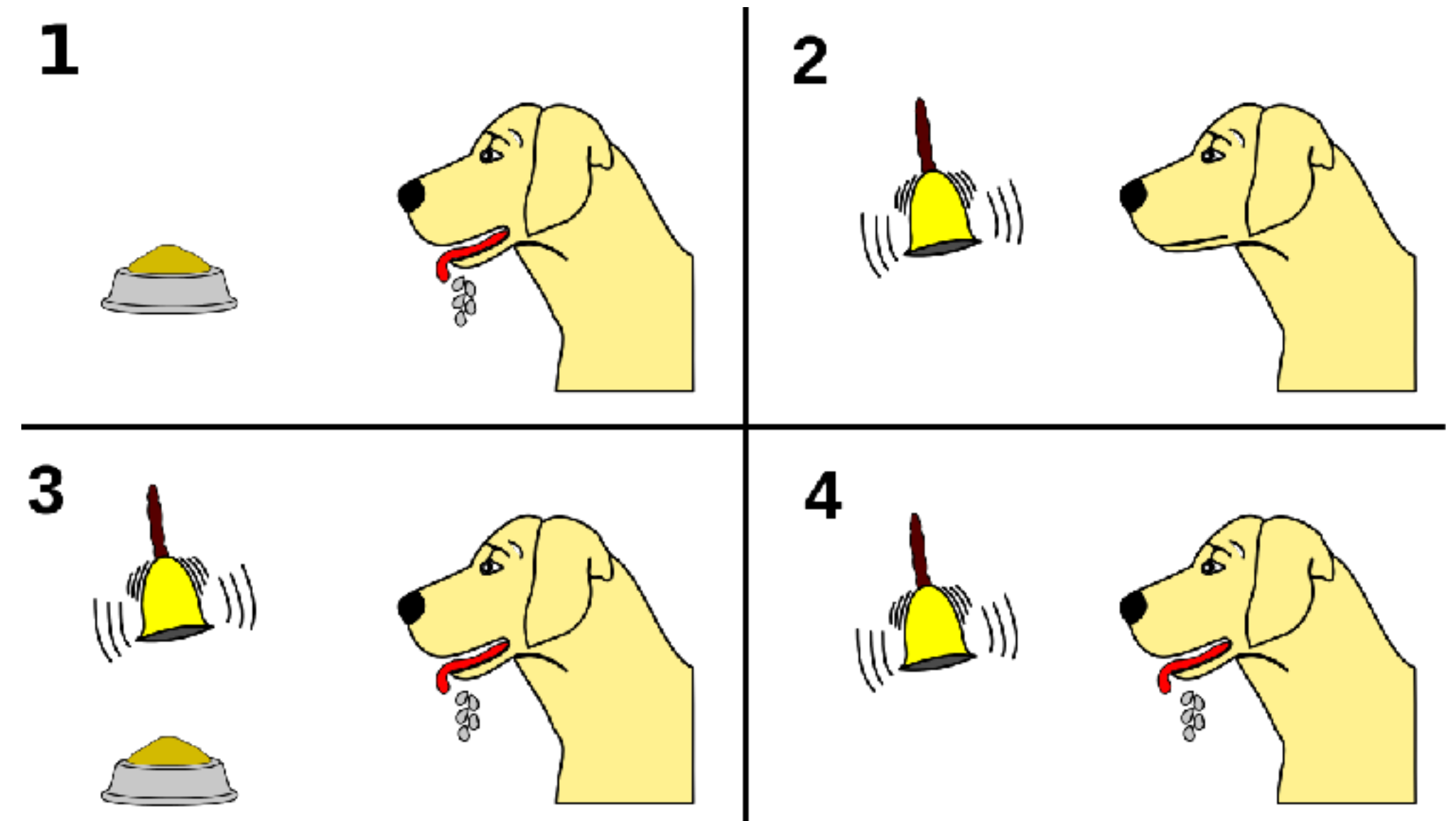
*Actions associated with satisfaction are strengthened, while those associated with discomfort become weakened.*



# Classical and Operant Conditioning

## Classical Condition (Pavlov, 1927)

Learning as the *passive* coupling of stimulus (bell ringing) and response (salivation), anticipating future rewards



## Operant Condition (Skinner, 1938)

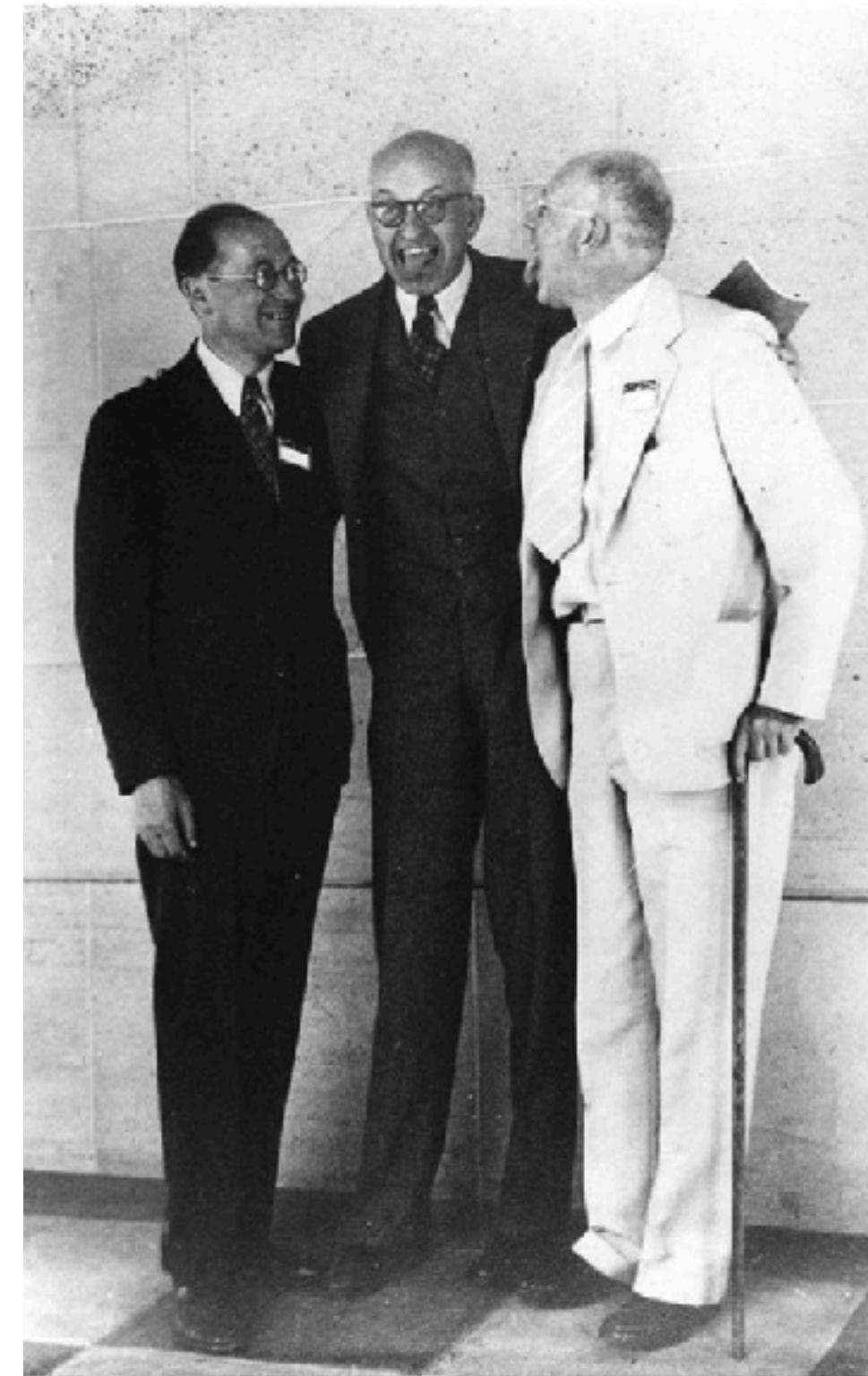
Skinner (1938): Learning as the *active* shaping of behavior in response to rewards or punishments





# Edward Tolman (1886 - 1959)

- Raised by an adamant Quaker mother
- Studied at MIT, Harvard, and Giessen
- Inspired by Gestalt psychologists like Kurt Koffka and Kurt Lewin
- Coined “*Purposive Behaviorism*”
  - Behavior needs to be studied in the context of the purpose or goals of behavior
- In contrast to other **behaviorists** at the time, Tolman believed in latent learning and the need to talk about hidden mental states in how we make decisions



Lewin, Tolman, & Hull

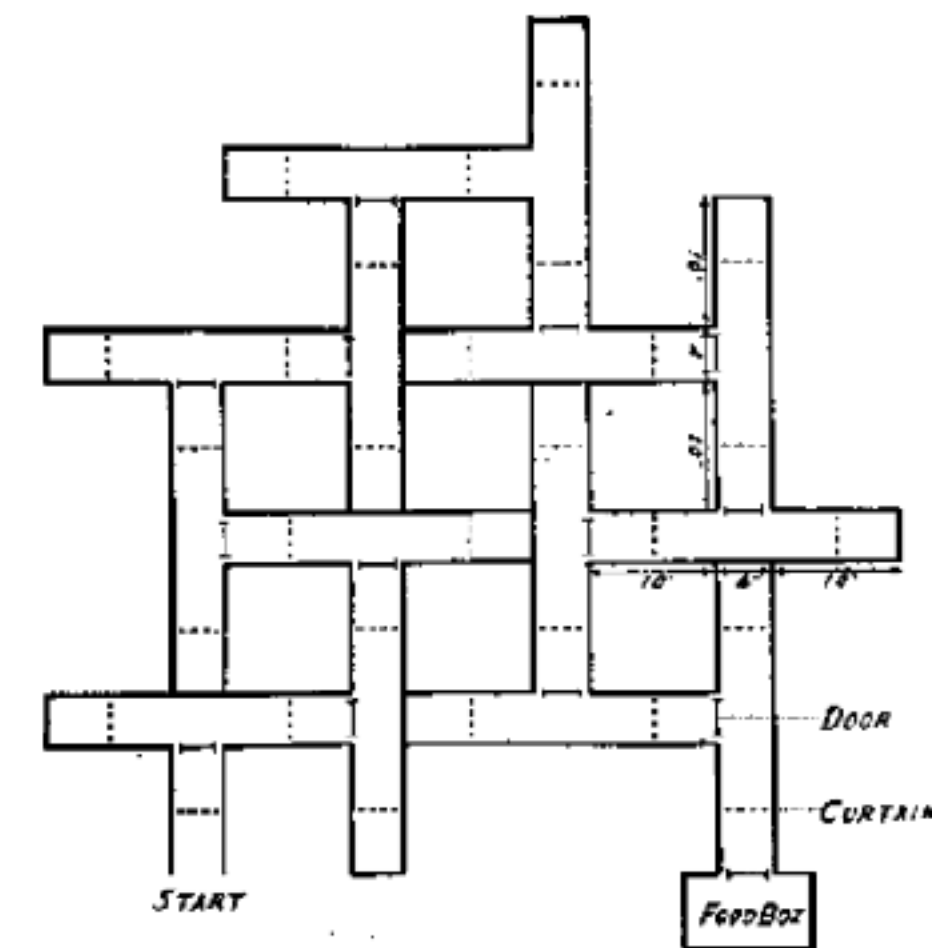
# Tolman and Cognitive maps

- Learning is not just a telephone switchboard connecting incoming sensory signals to outgoing responses (S-R Learning)
- Rather, “latent learning” establishes something like a “field map of the environment” gets established (S-S learning)

## Stimulus-Response (S-R) Learning



## Stimulus-Stimulus (S-S) Learning



Plan of maze  
11 Unit T Alley Maze

FIG. 1

(From M. H. ELLIOTT, The effect of change of reward on the maze performance of rats. *Univ. Calif. Publ. Psychol.*, 1928, 4, p. 20.)

# Tolman (1948): Different interpretations

*“All students agree as to the facts. They disagree, however on theory and explanation”*

- **S-R school:** learning consists of strengthening/weakening of S-R connections (like a telephone exchange)
  - subgroup a) more frequent responses are strengthened (Law of Exercise)
  - subgroup b) more rewarded responses are strengthened (Law of Effect)
- **S-S school:** in the course of learning, *“a field map of the environment gets established”*
  - Sampling of stimuli is not passive, but active and selective during learning w.r.t. to a goal or purpose
  - Stimuli are not just routed to associations, but used to construct some new map-like representation that captures the relational structure of the environment
  - The nature of these map-like representations (strip-like vs. broad) have consequences for generalization



# Experiments

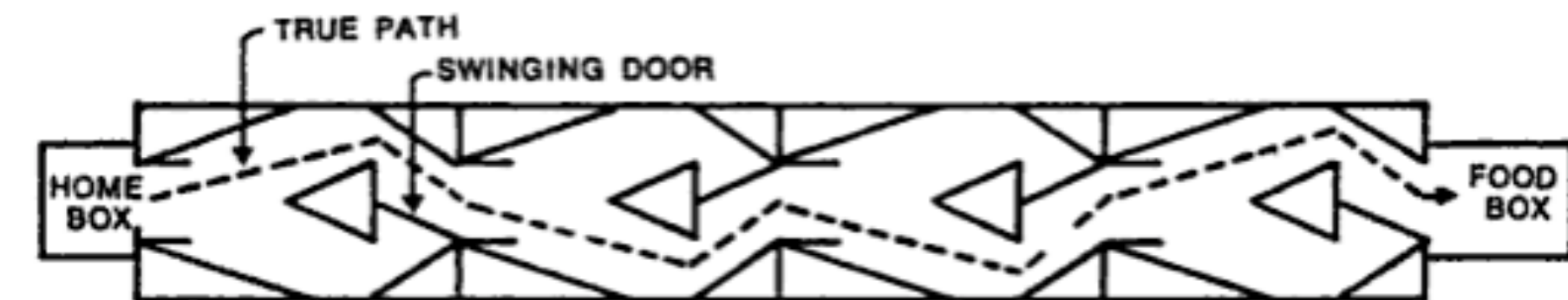
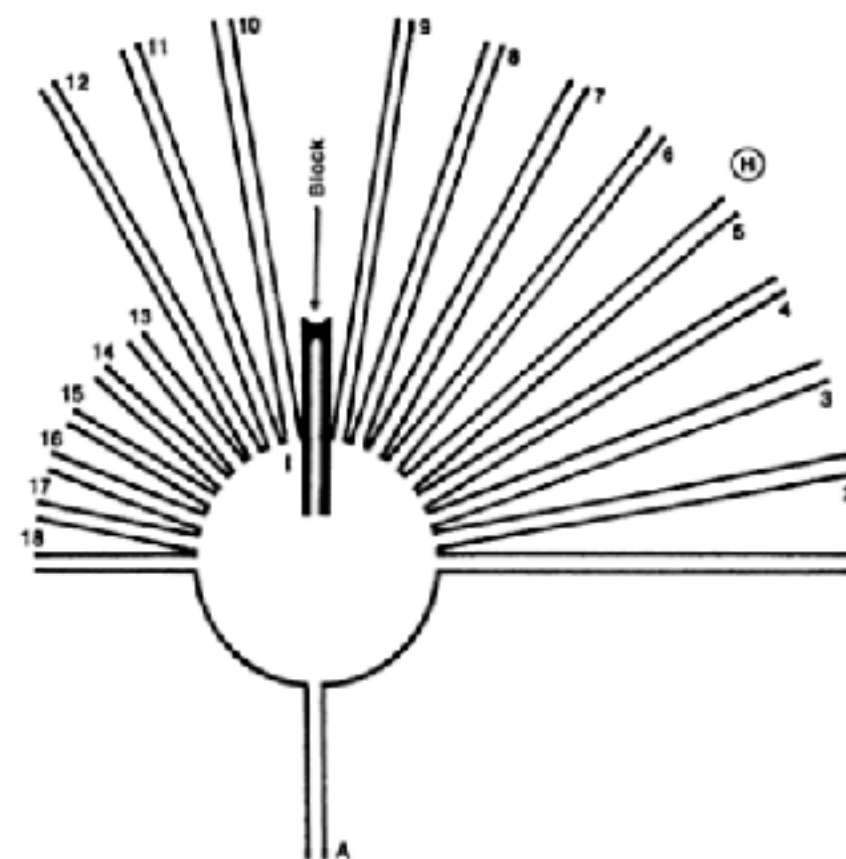
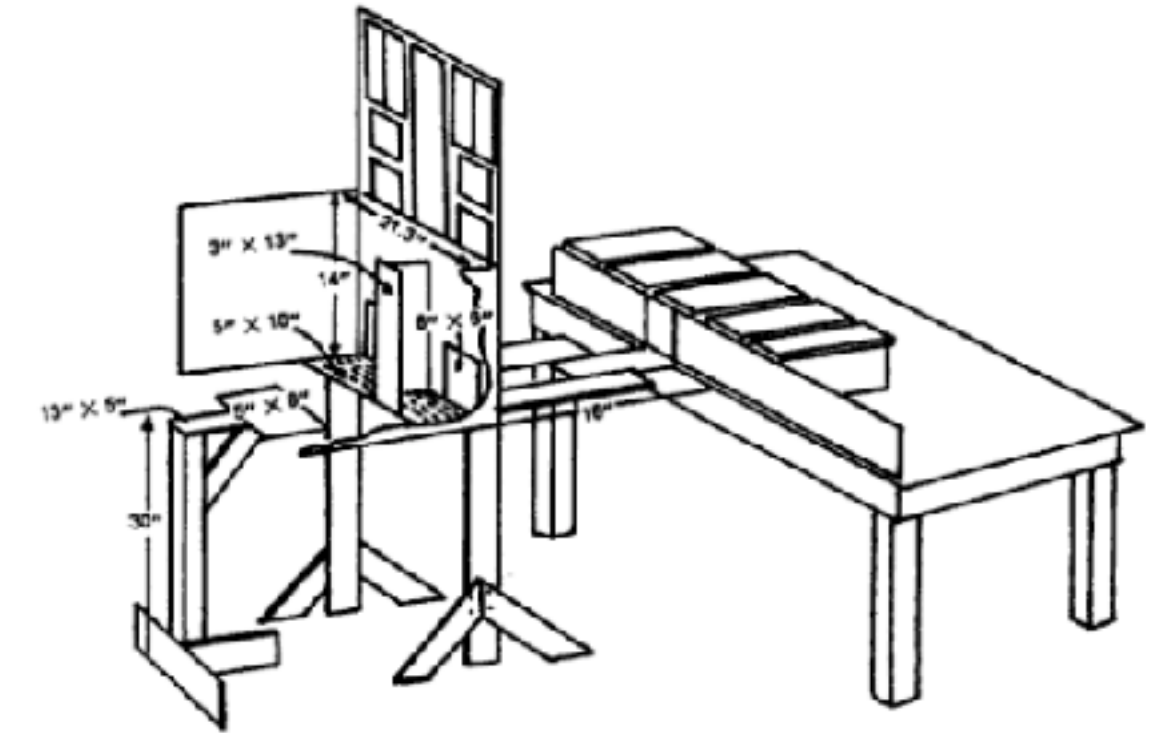
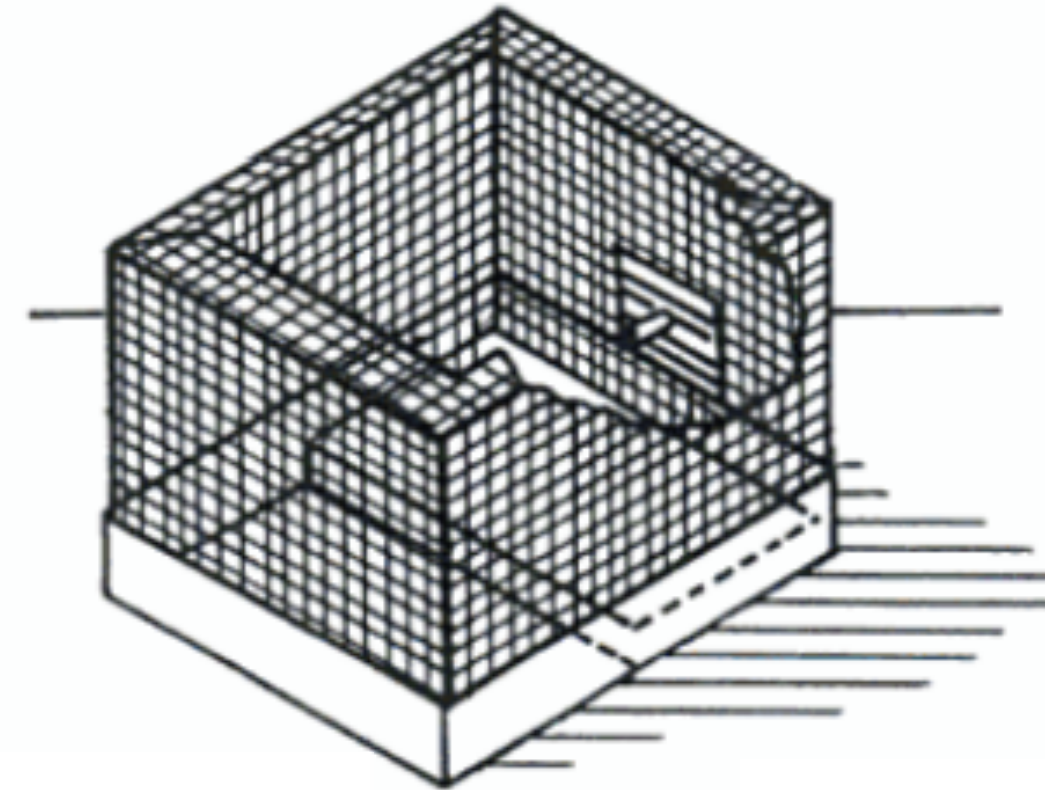
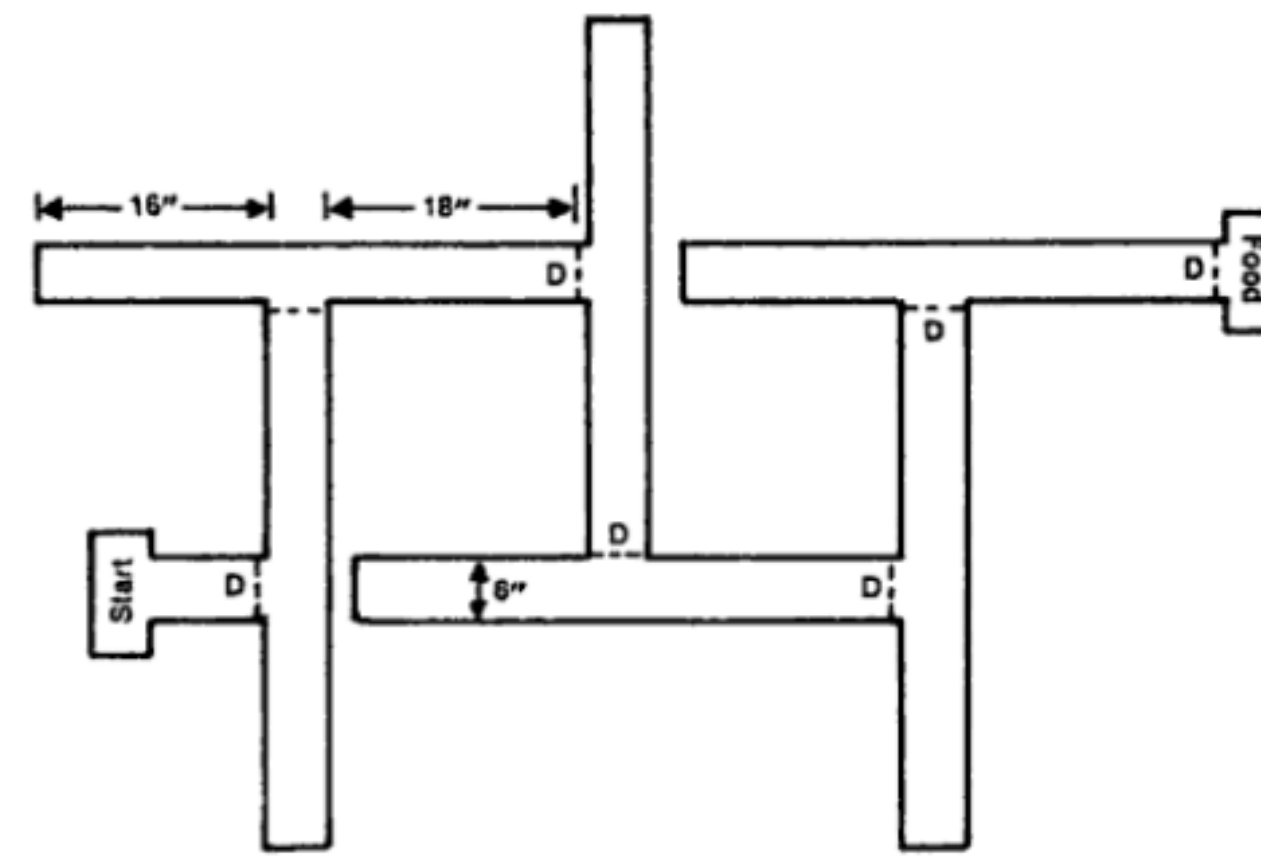
1. Latent Learning

2. Vicarious trial and error

3. Searching for the stimulus

4. Hypotheses

5. Spatial orientation



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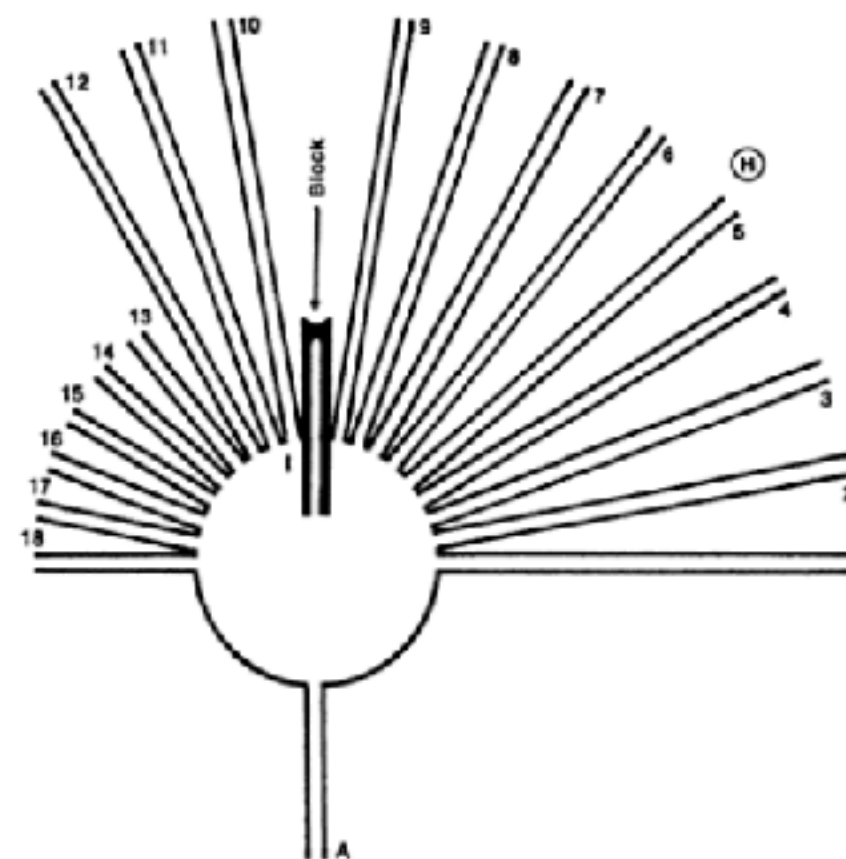
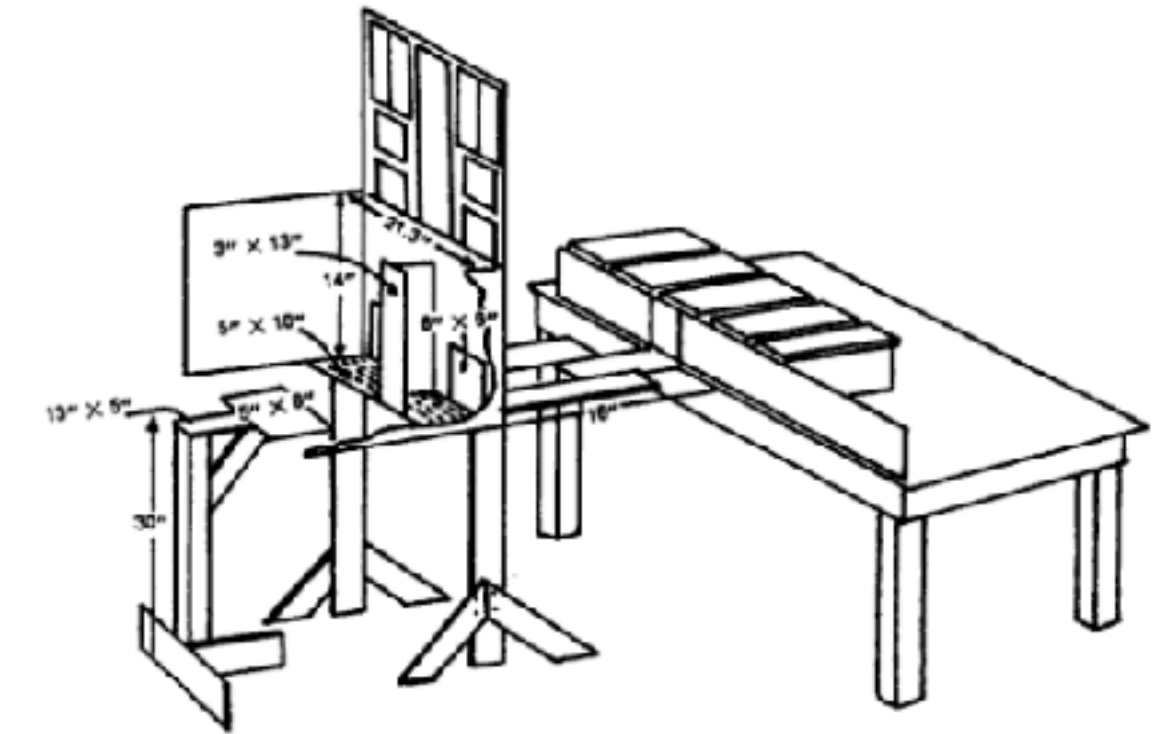
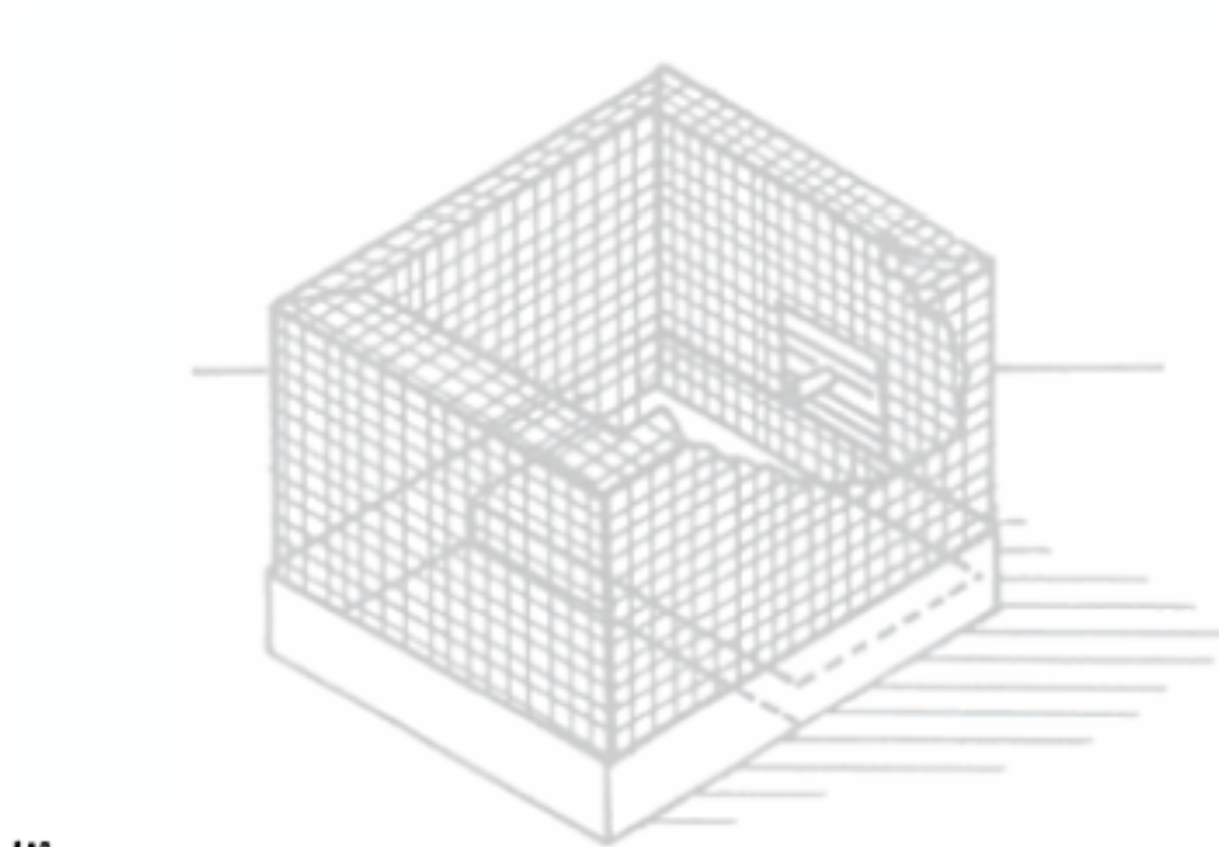
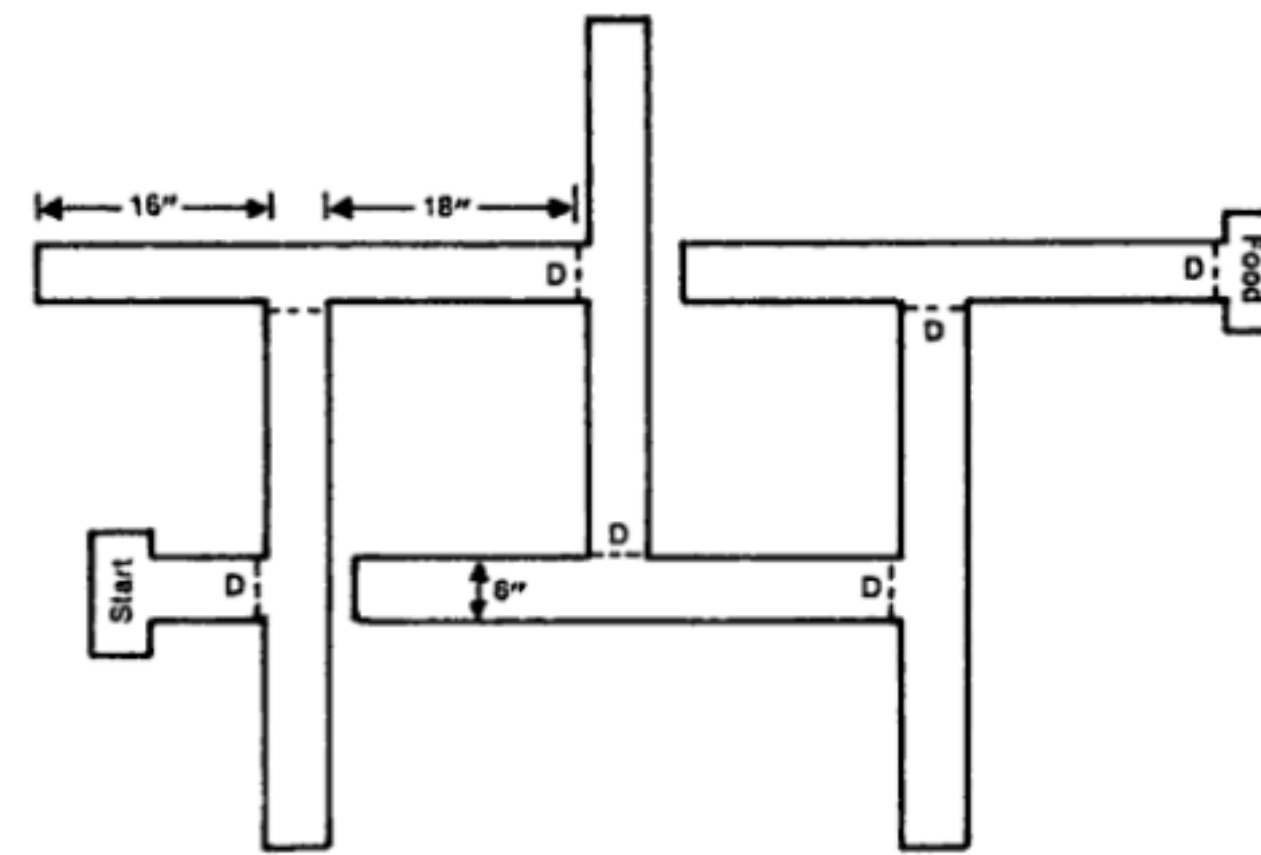
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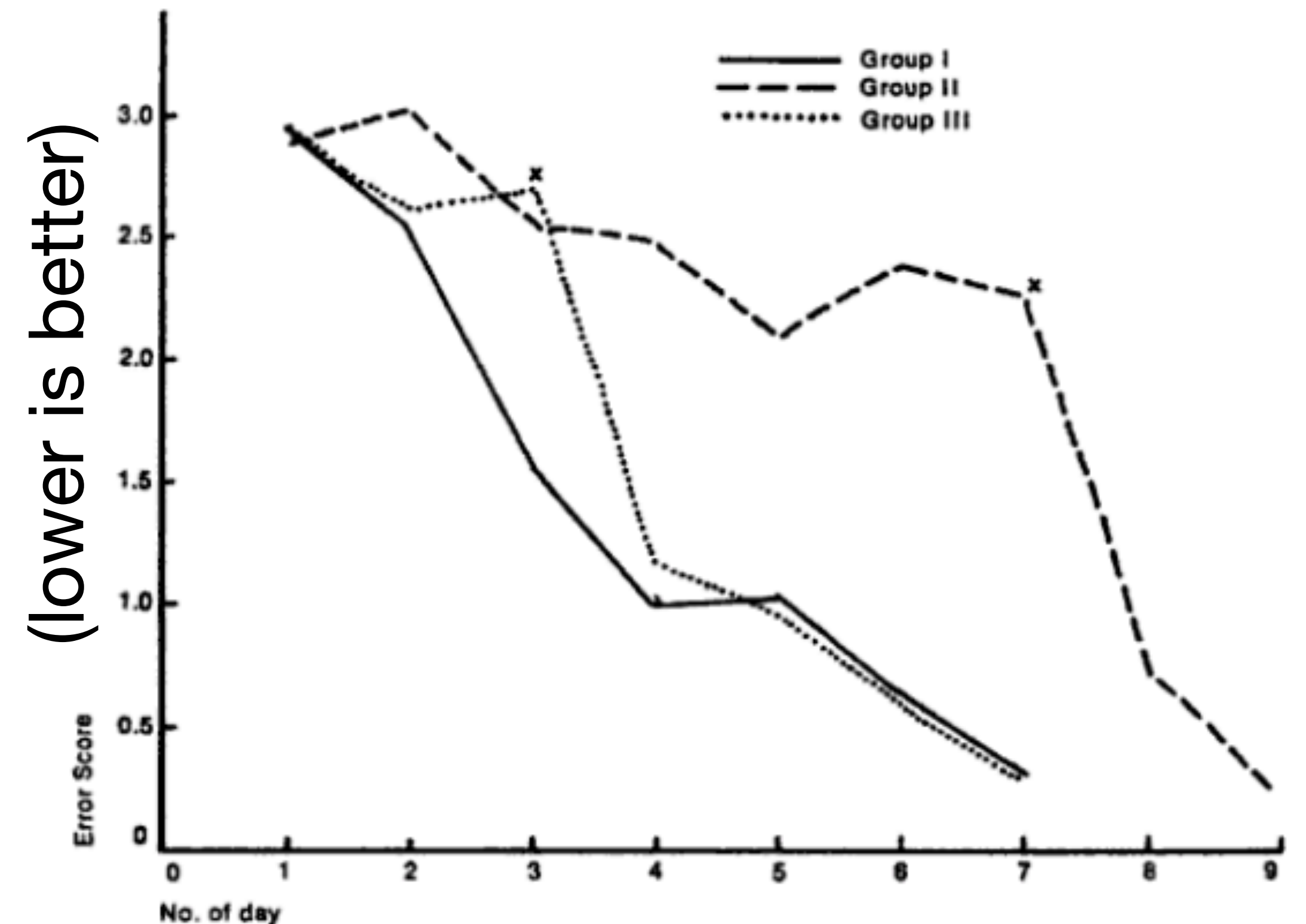
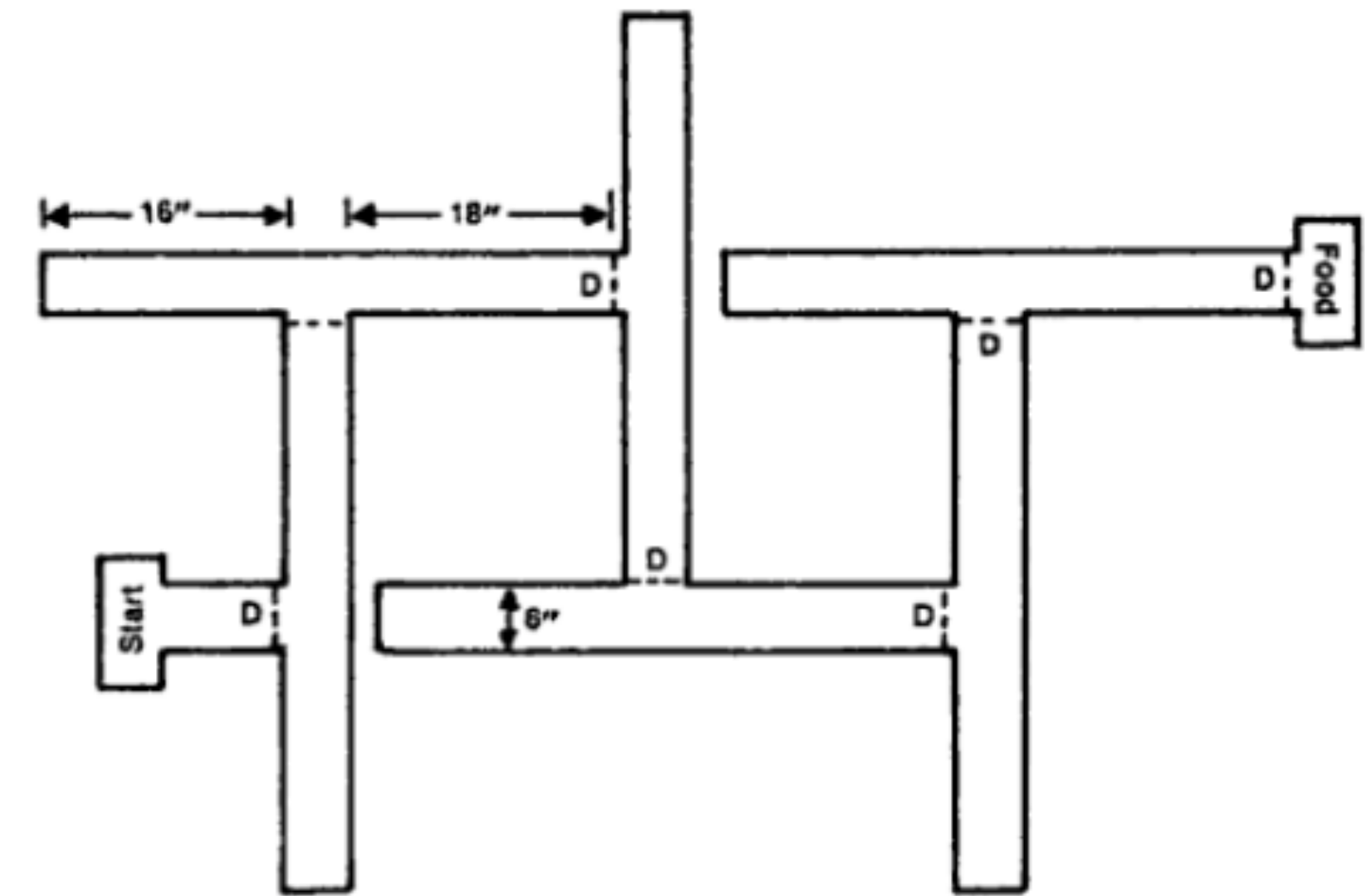
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# Latent Learning

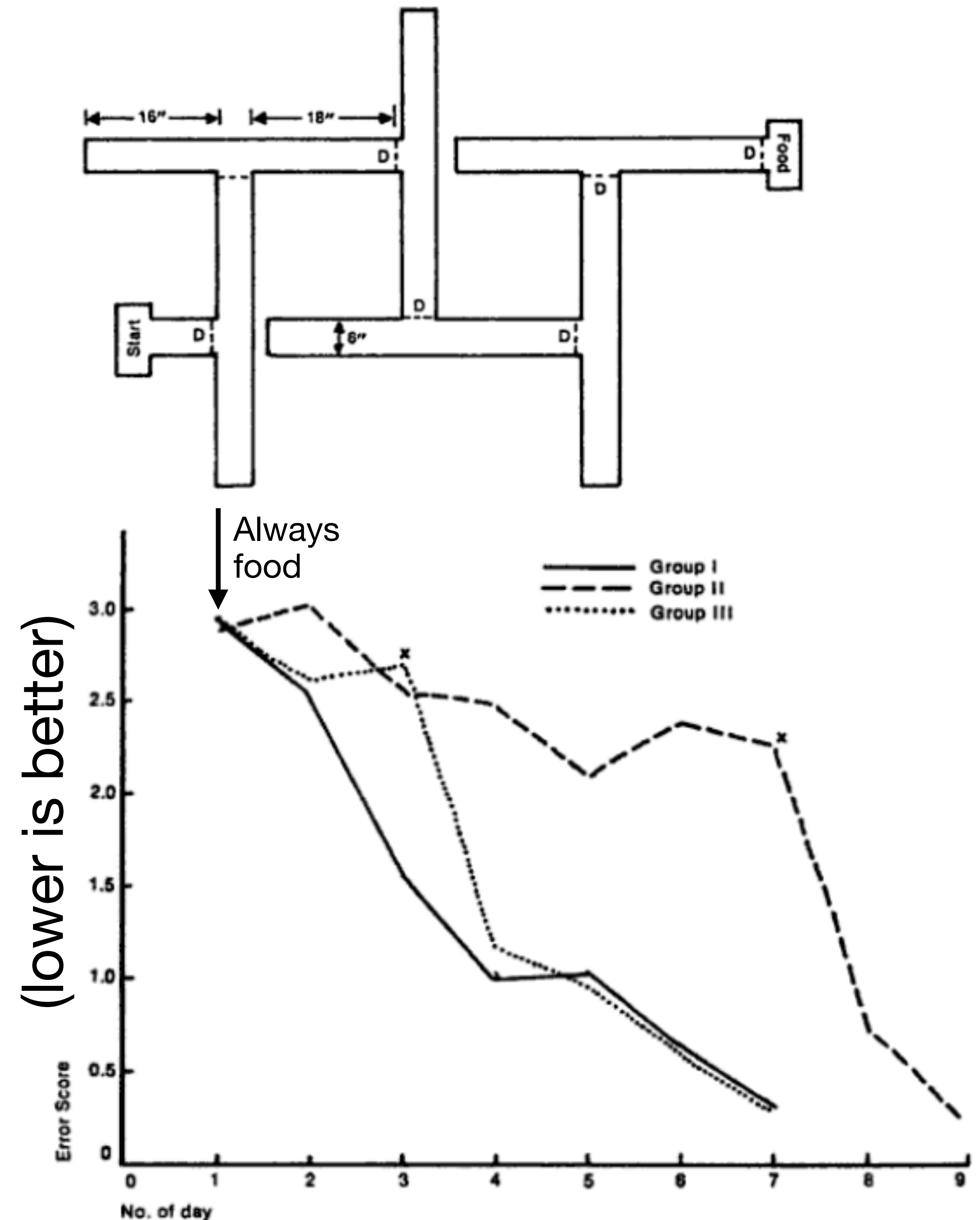
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- Learning curves dropped dramatically when food was added
  - This suggests latent learning prior to reward
  - “They had been building up a ‘map’”
  - Once the reward was added, they could use the map rather than starting from scratch





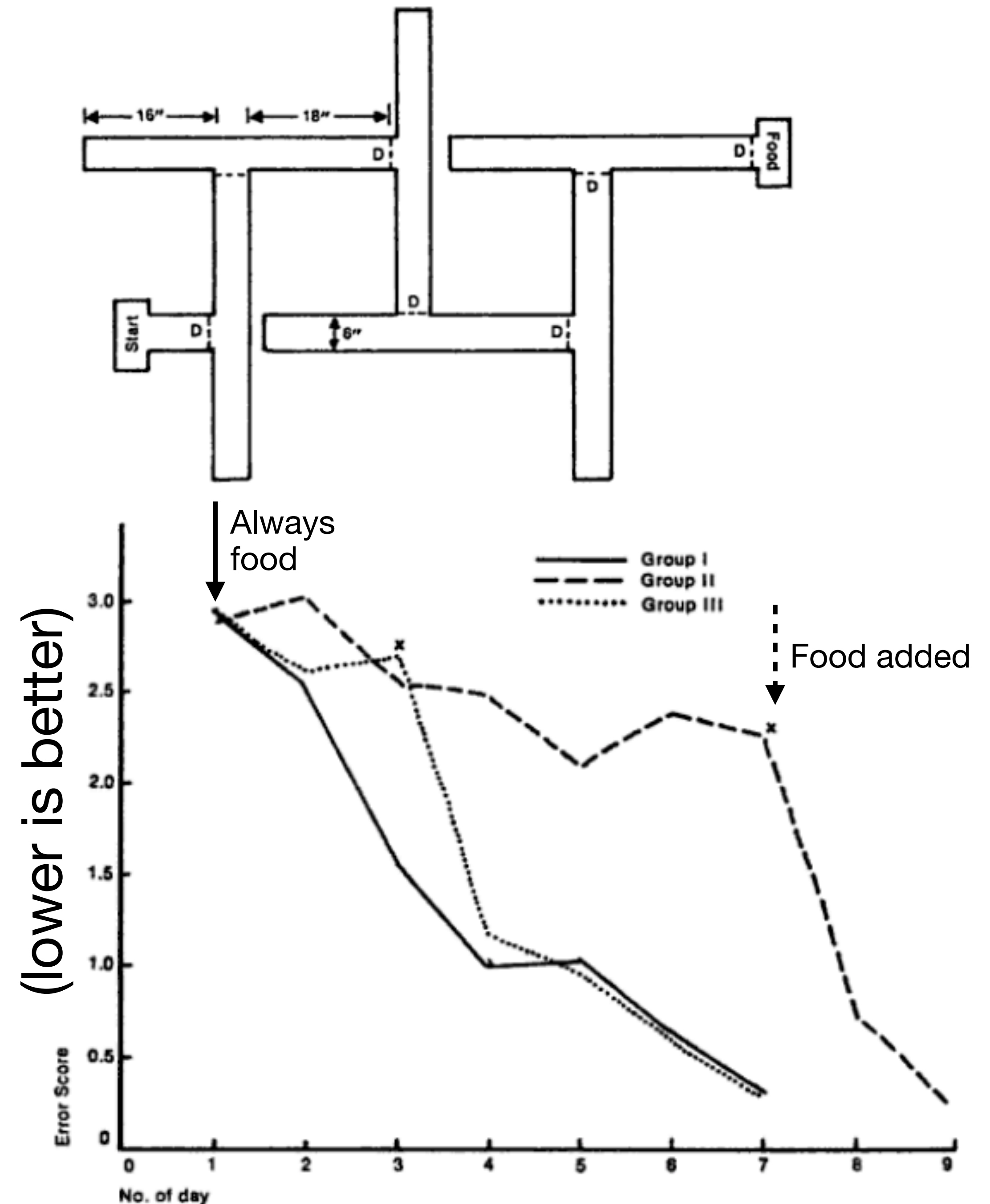
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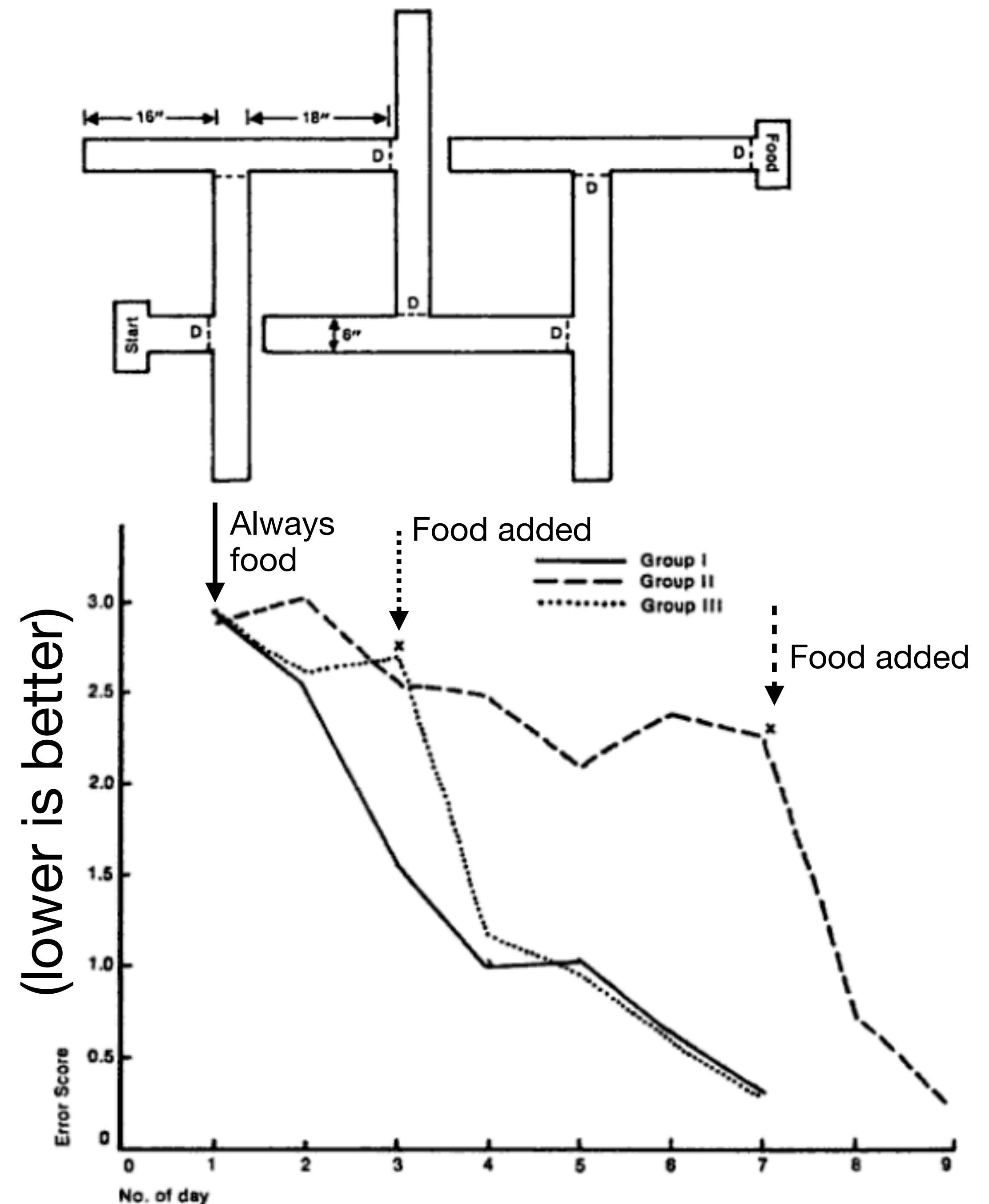
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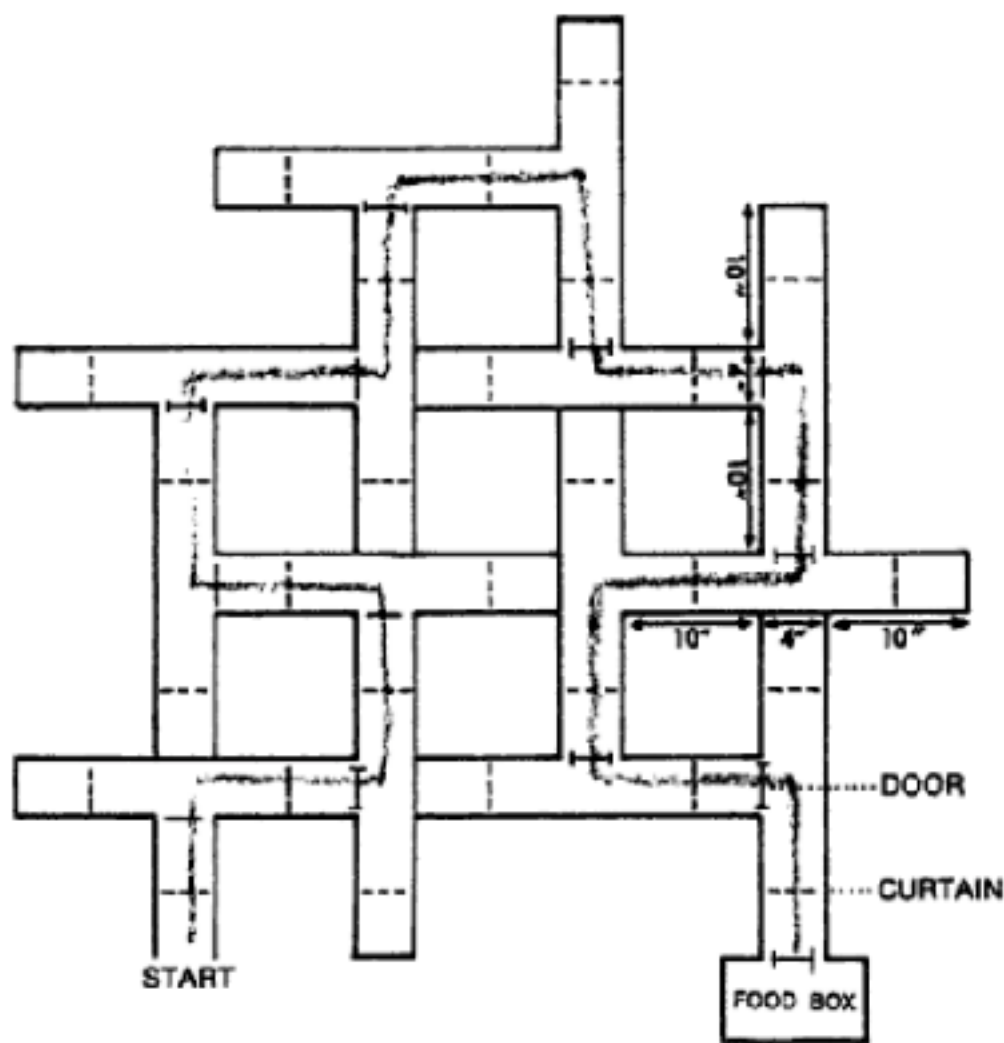
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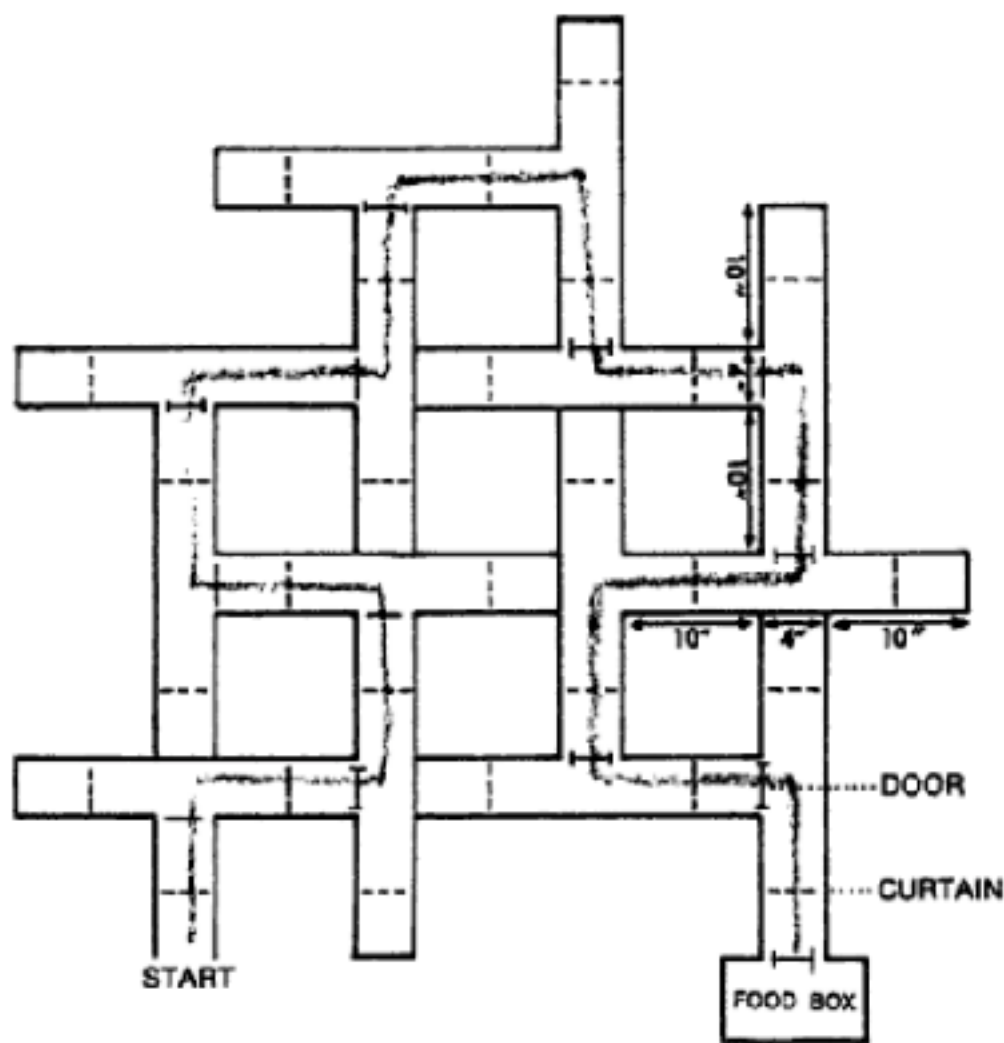
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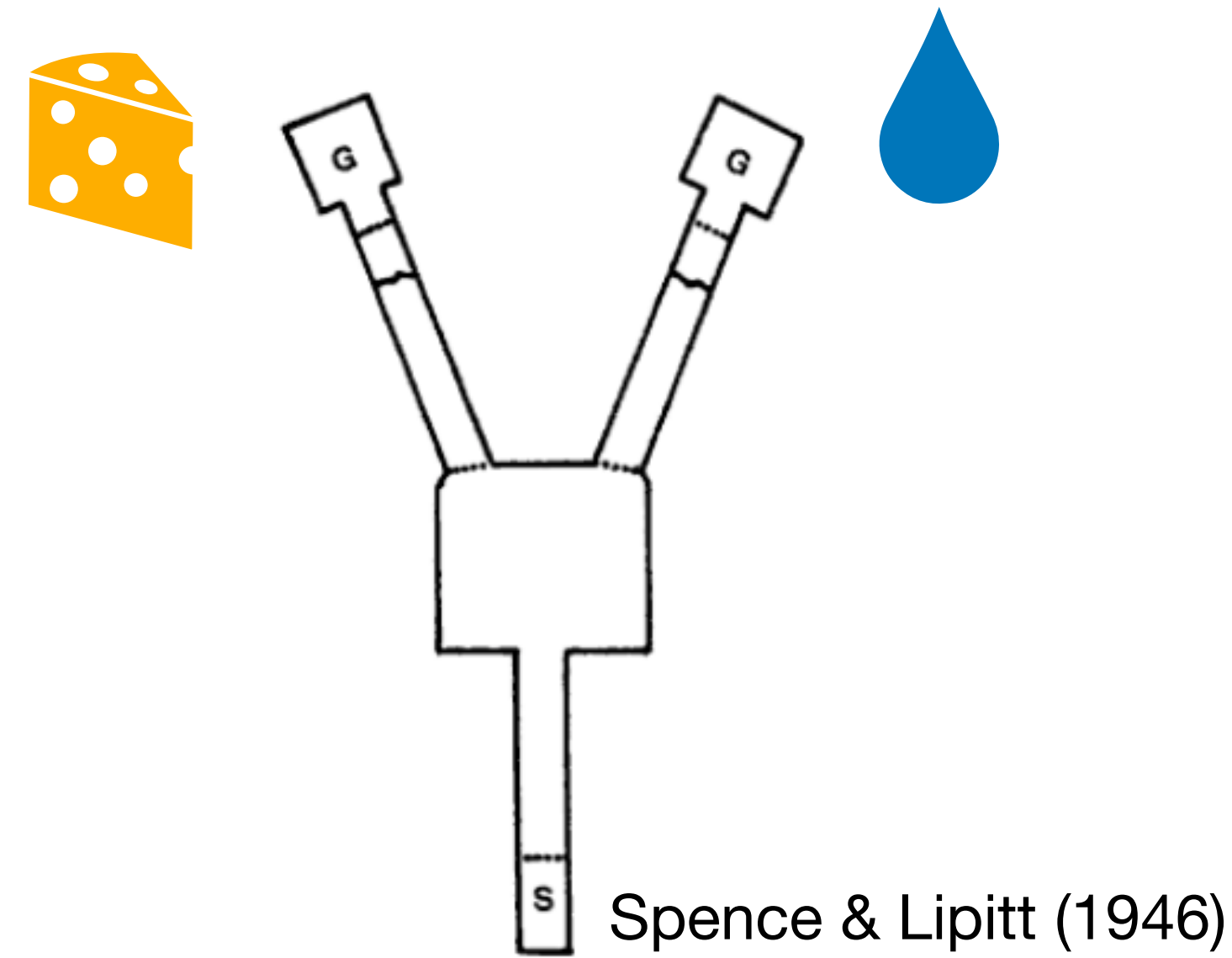
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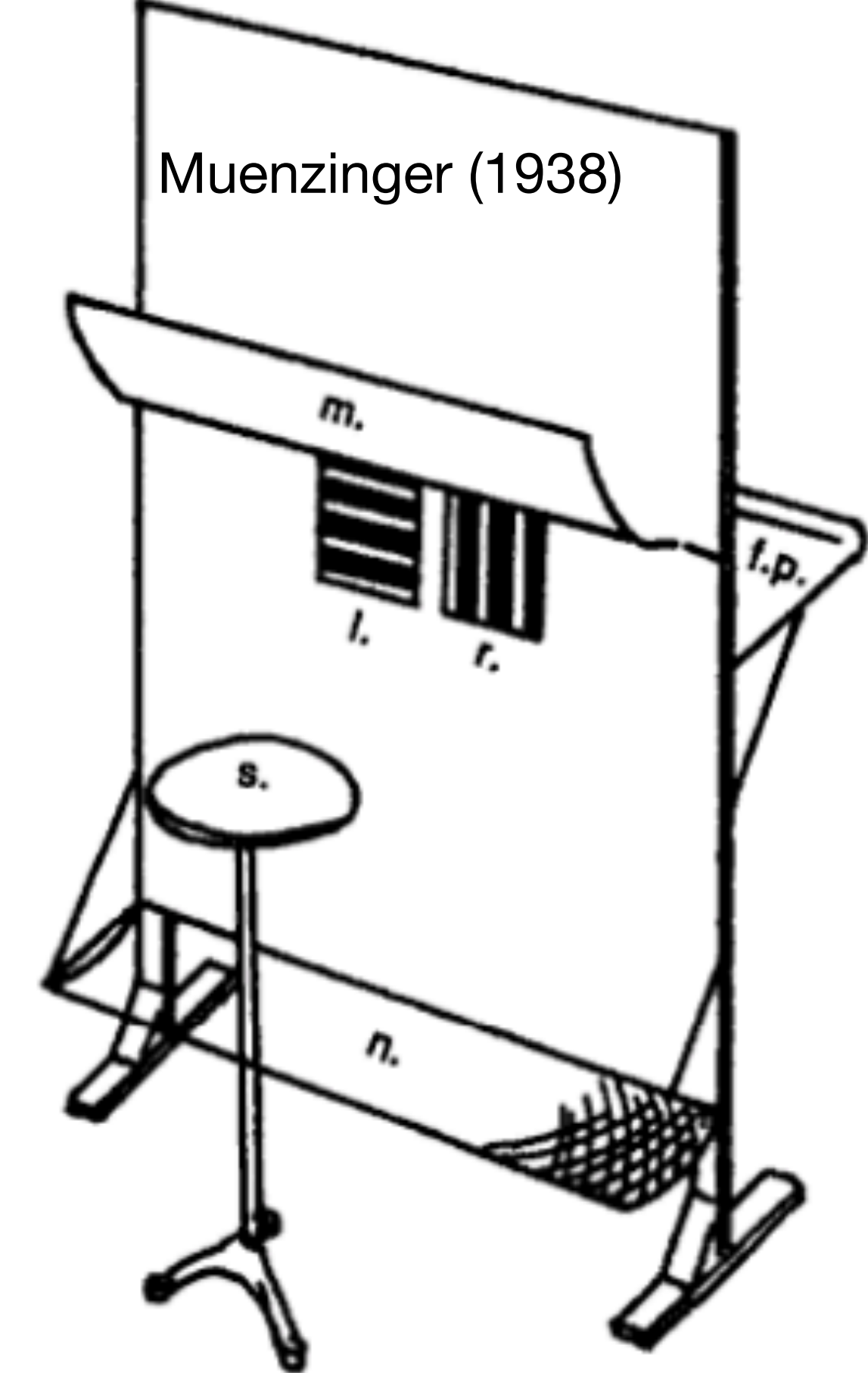
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- **Y-maze** with separate food 🧀 + water 💧 rewards
- Rats exposed to maze while satiated (no **hunger** + no **thirst**)
- One group reintroduced when **hungry** goes left towards 🧀
- Another group reintroduced when **thirsty** goes right towards 💧

# Vicarious Trial and Error (VTE)

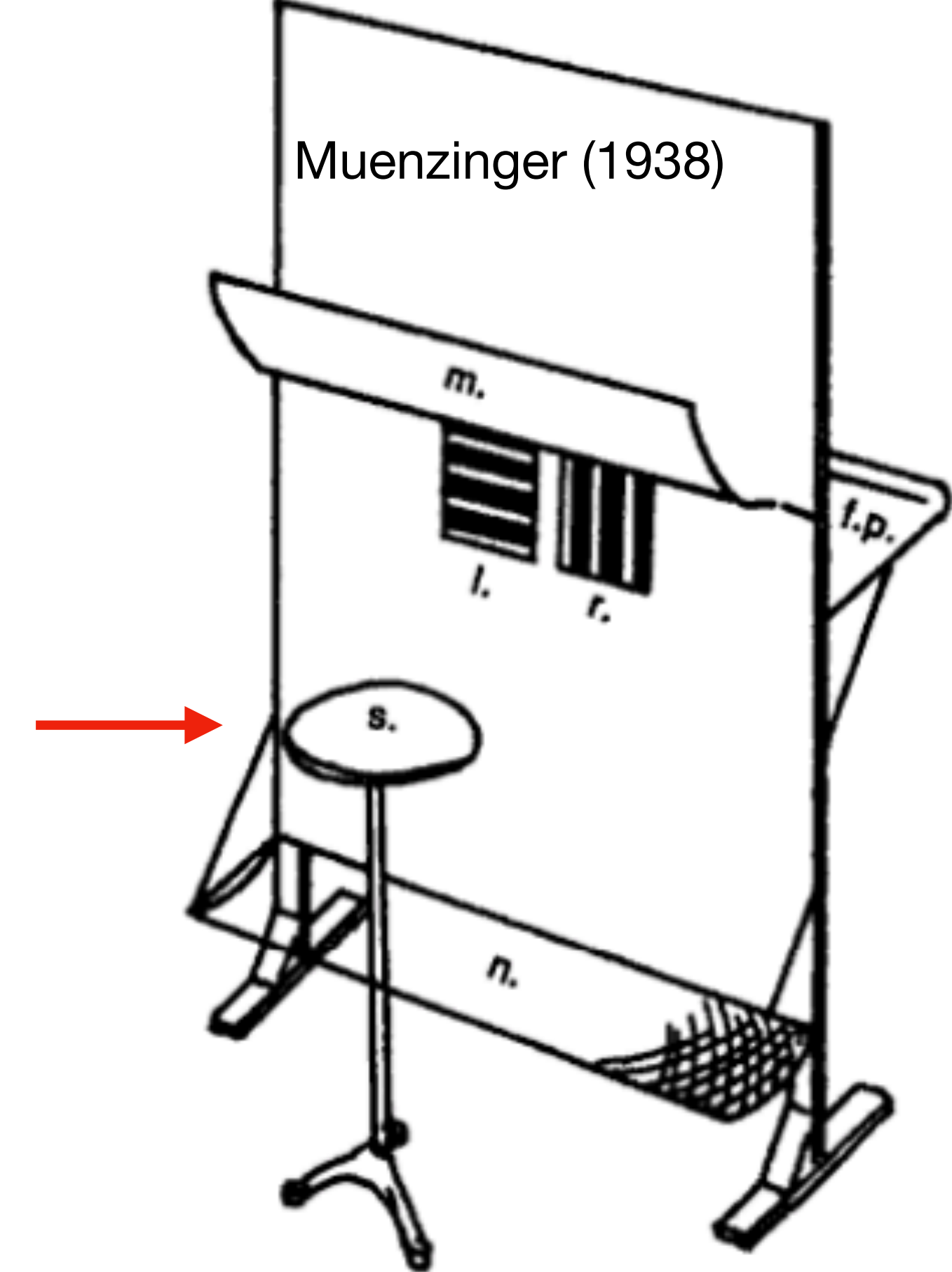
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  - location is randomly swapped but visual features are predictive
  - If the animal jumps towards the correct door, it opens and reveals food on a platform behind... and if incorrect ....
- Tolman (1939) added landing platforms in front of the doors
  - When the choice was easy (black vs. white stimuli), the animals learned quicker and did more VTEing than for hard problems
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  - Better learners also did more VTEing (Geier, Levin & Tolman, 1941)





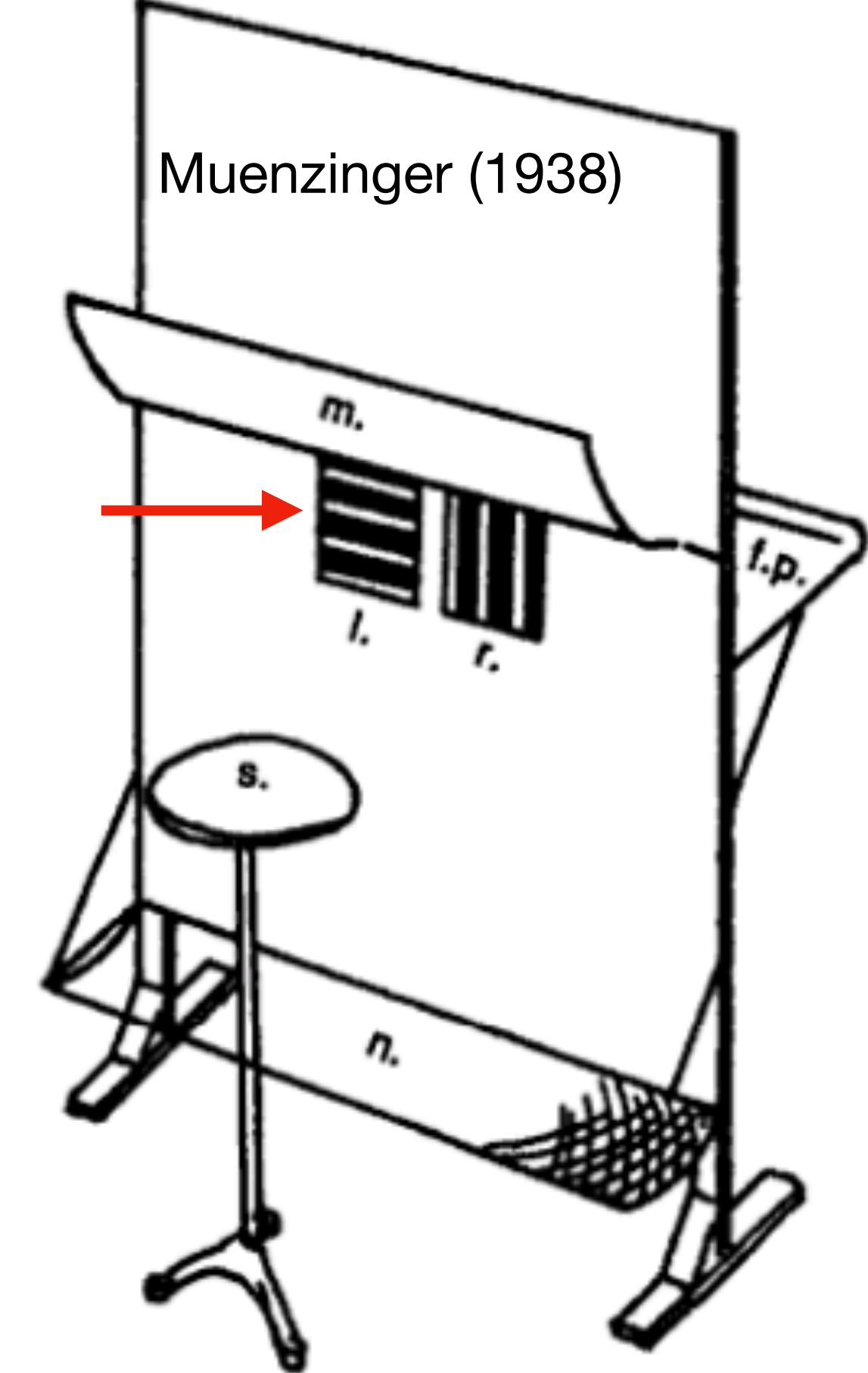
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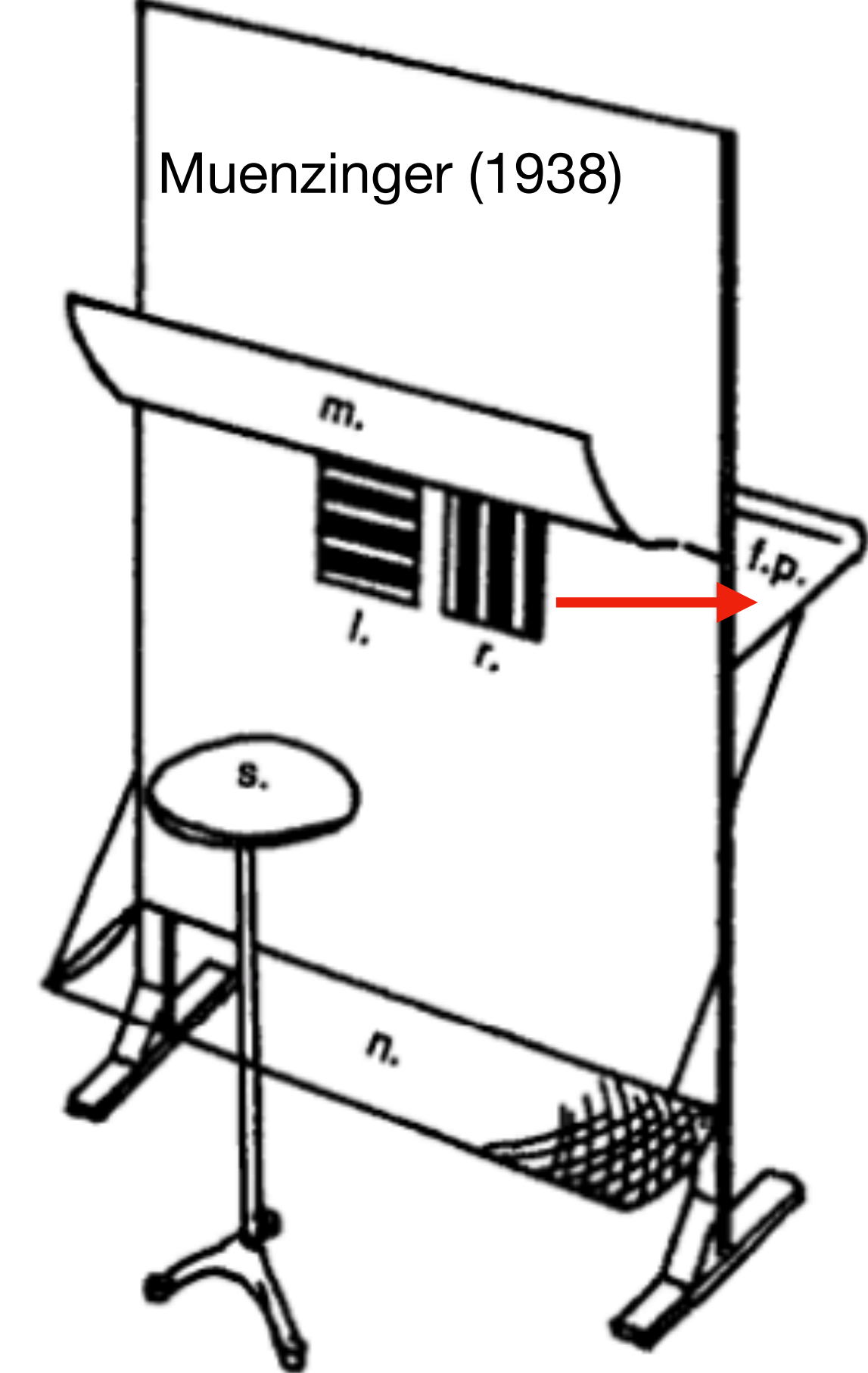
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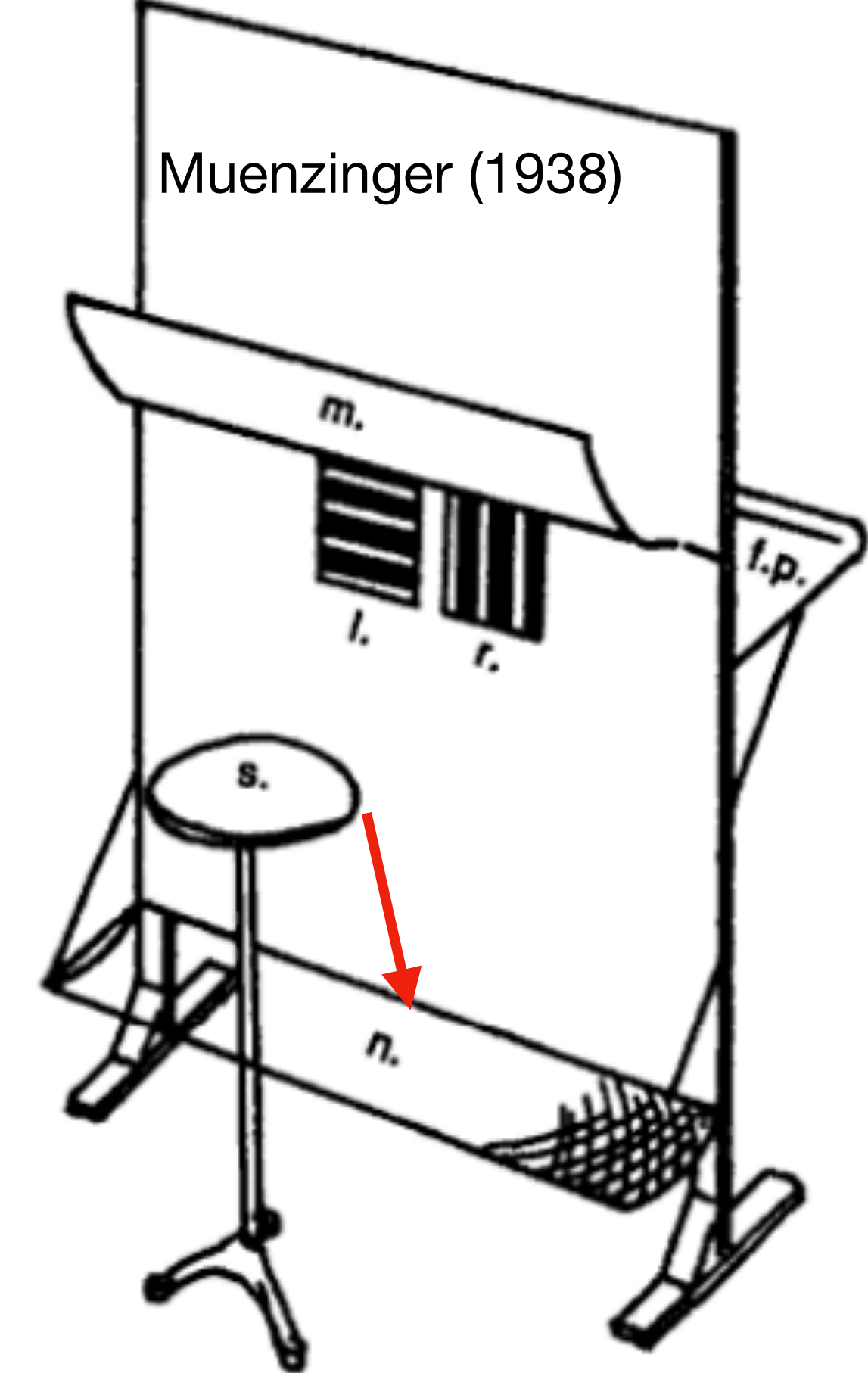
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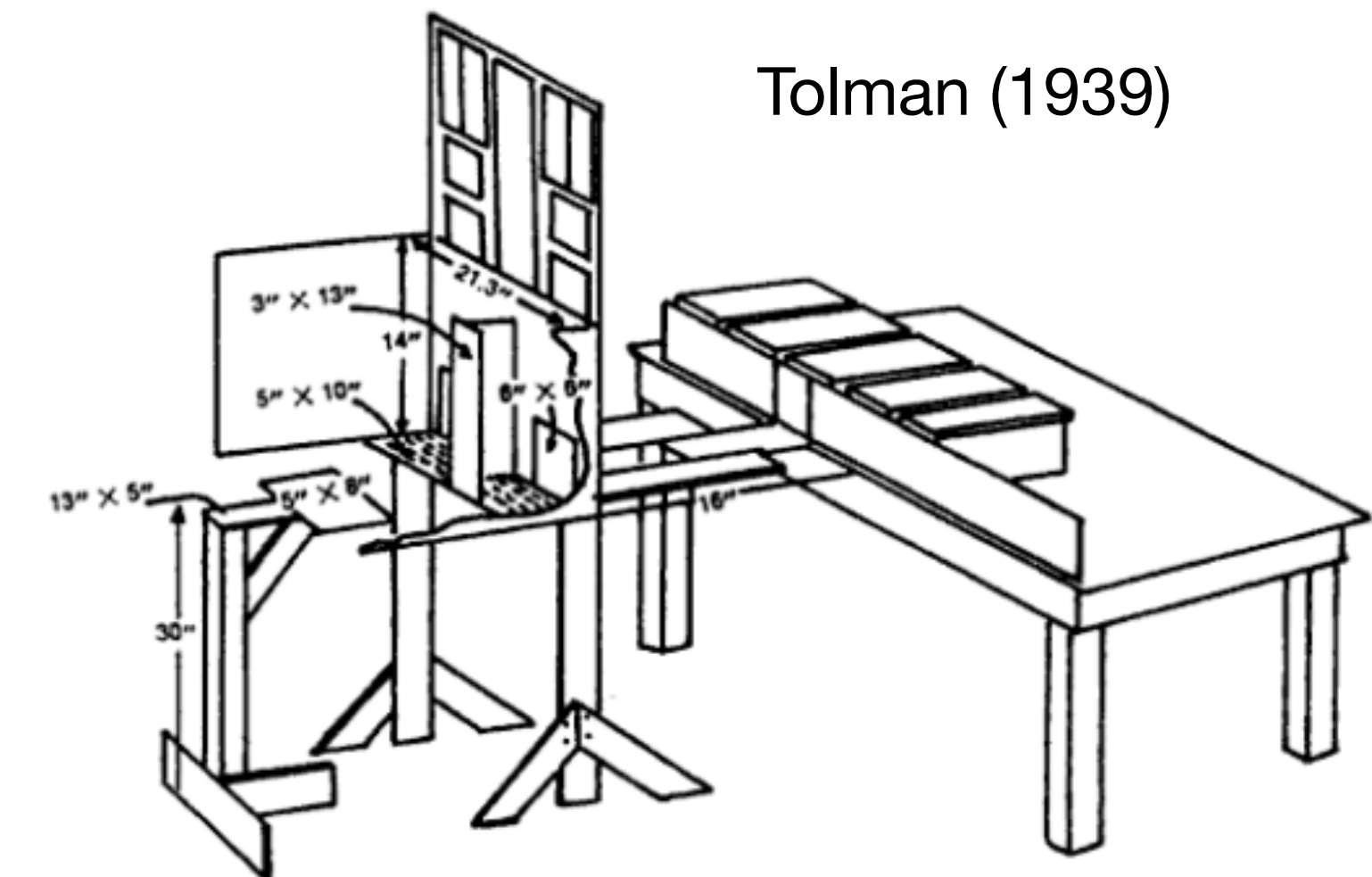
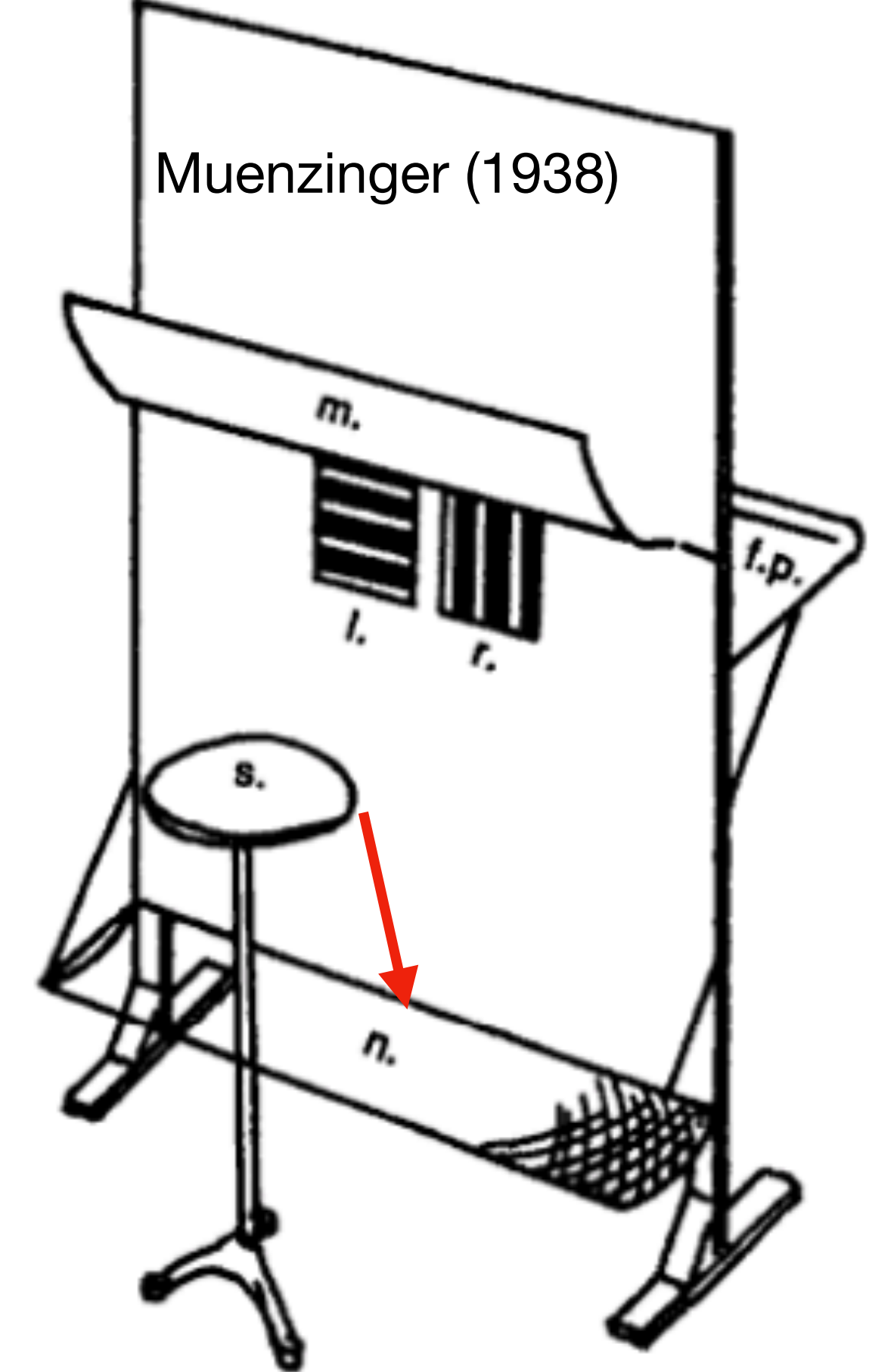
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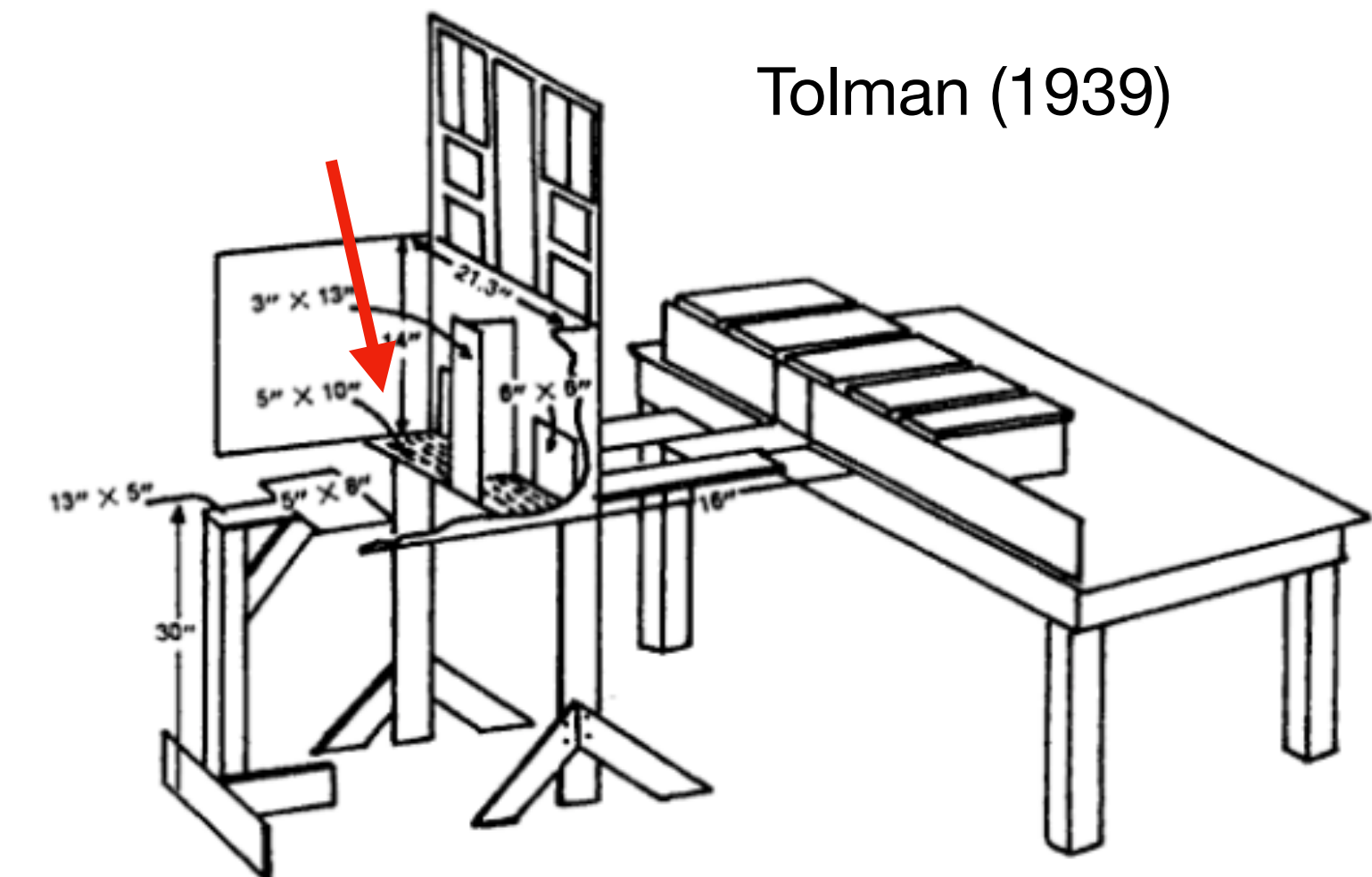
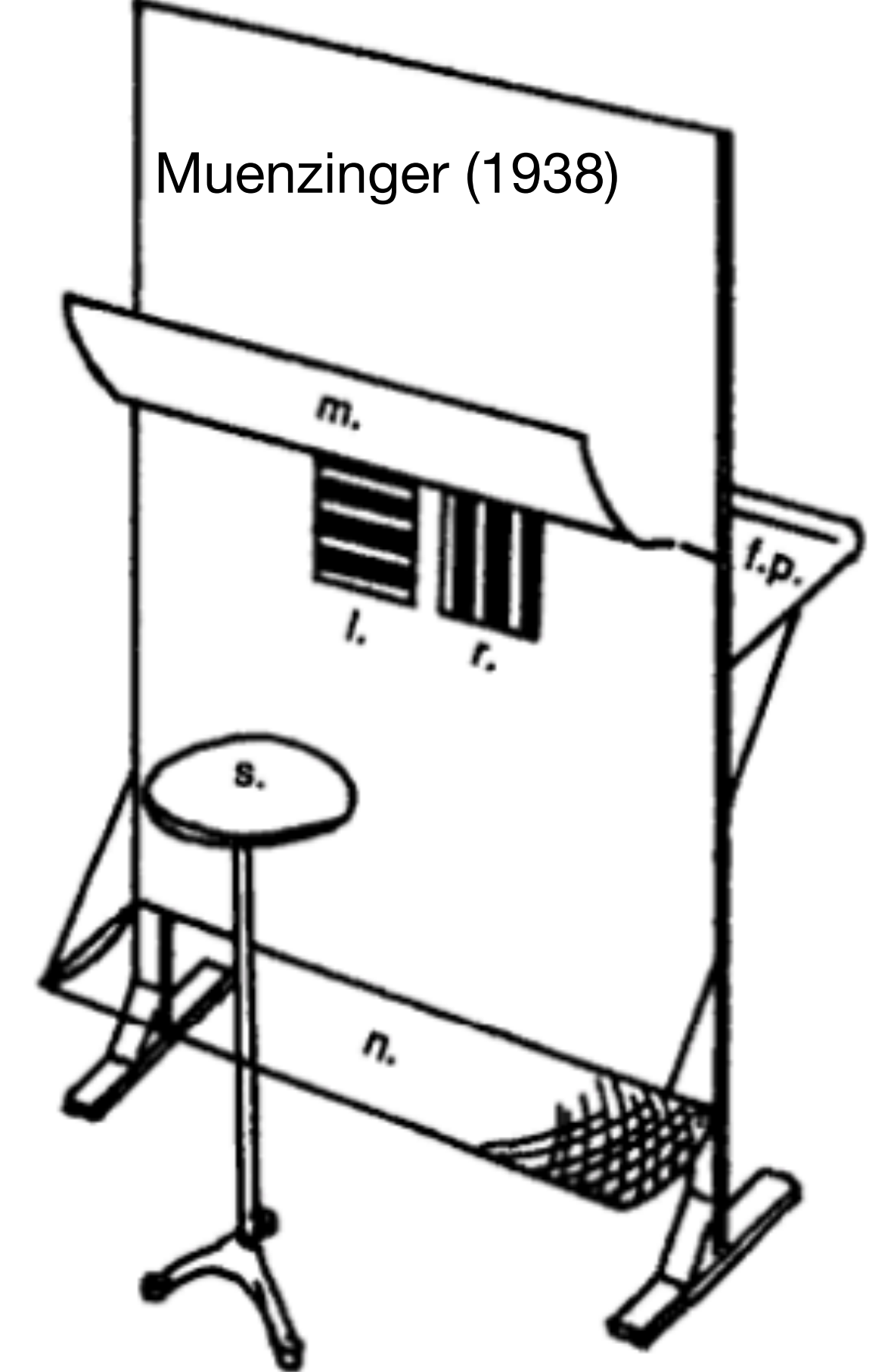
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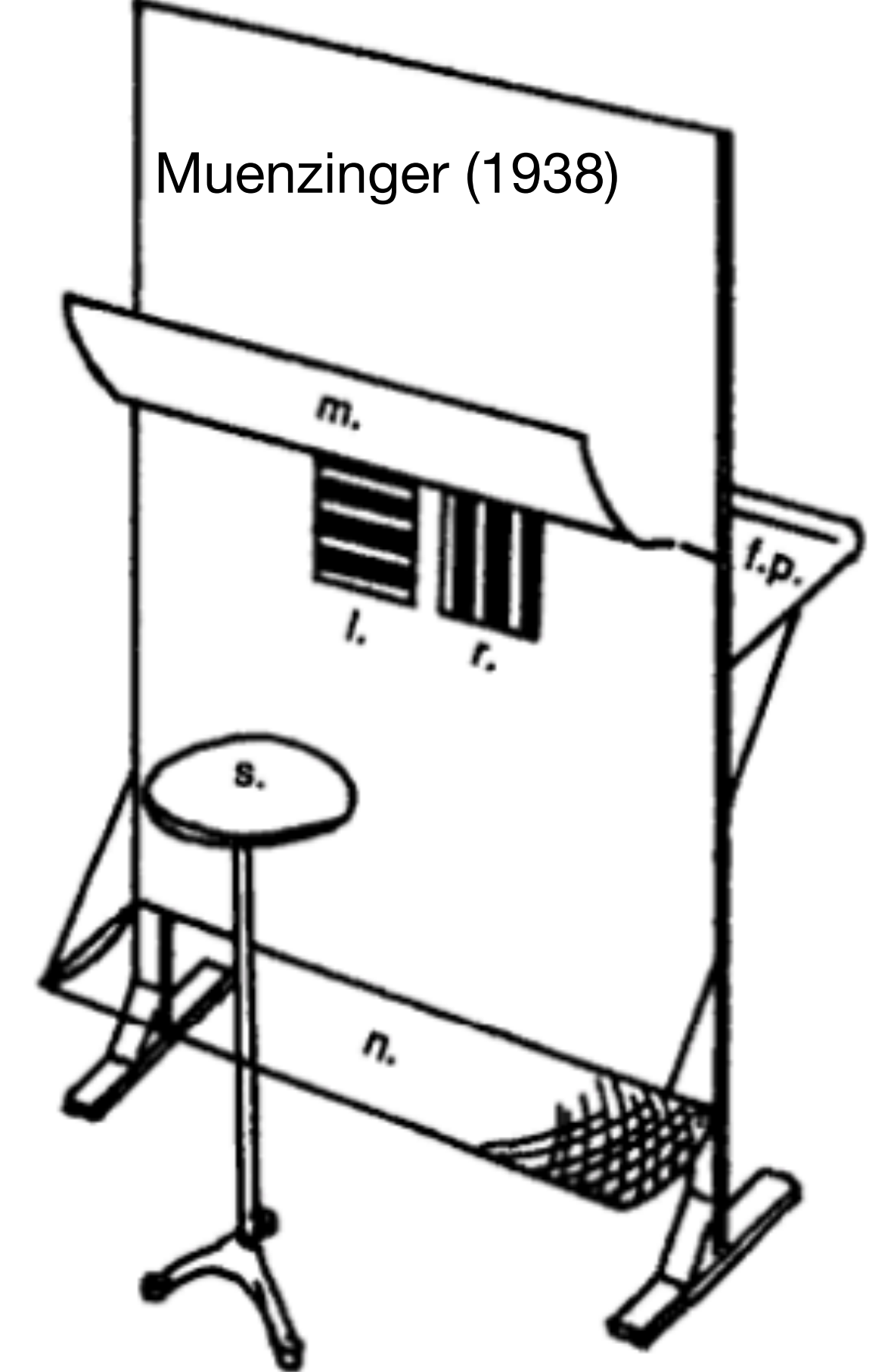
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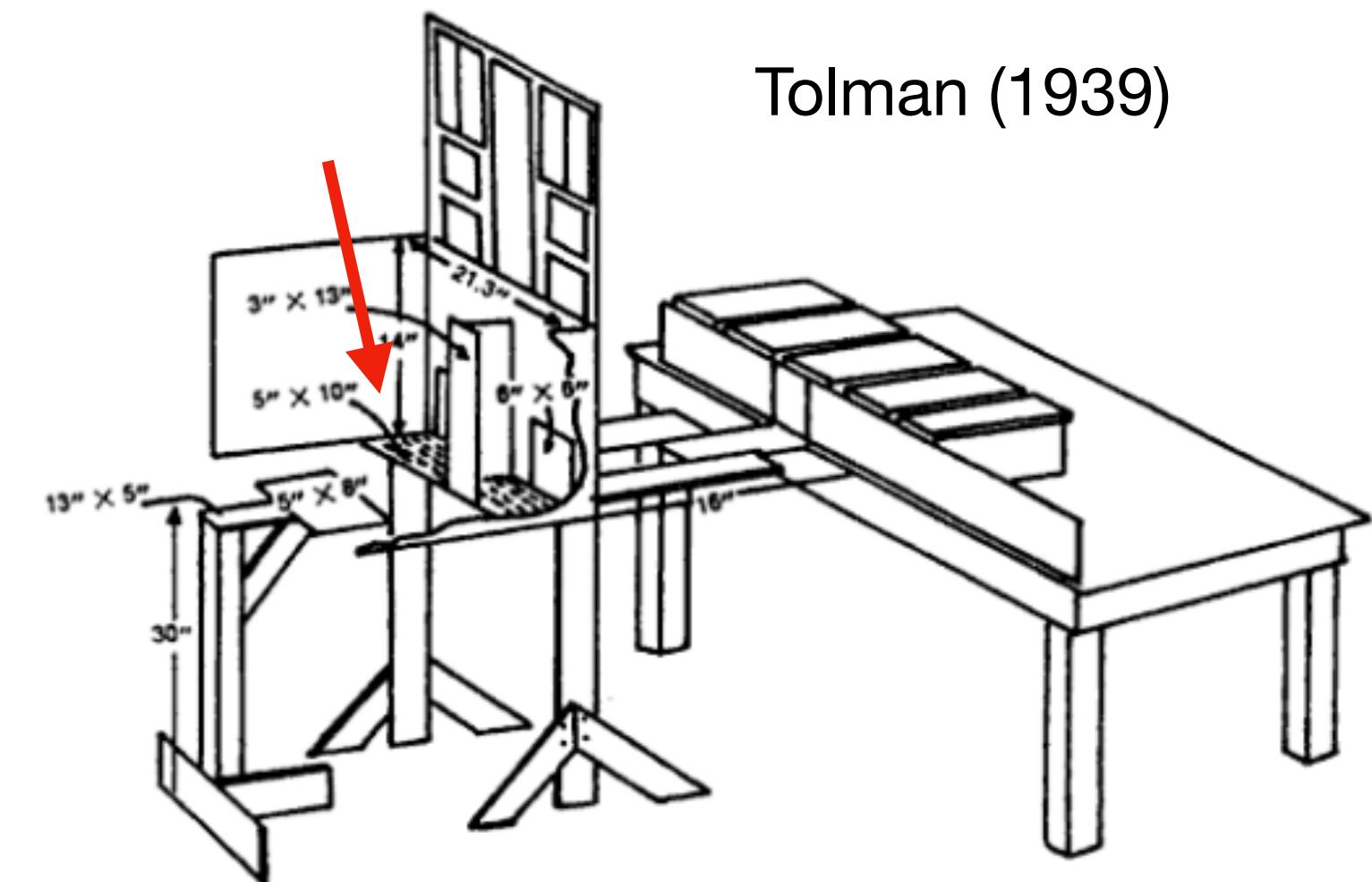


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Tolman (1939)



**Vicarious trial and error (VTE):** hesitating, looking-back-and-forth behavior observed in rats when confronted with a choice





AVIOLI P...



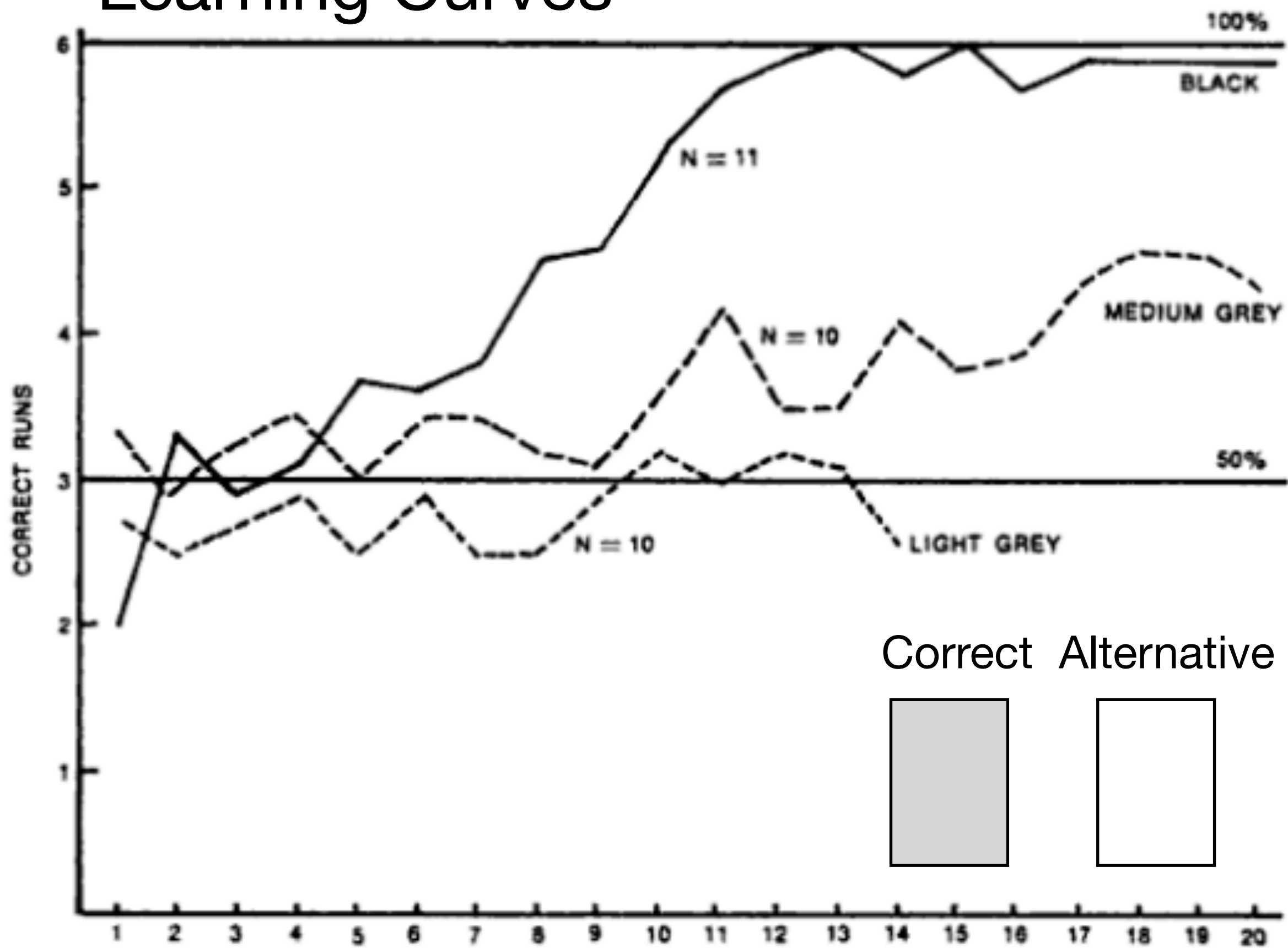




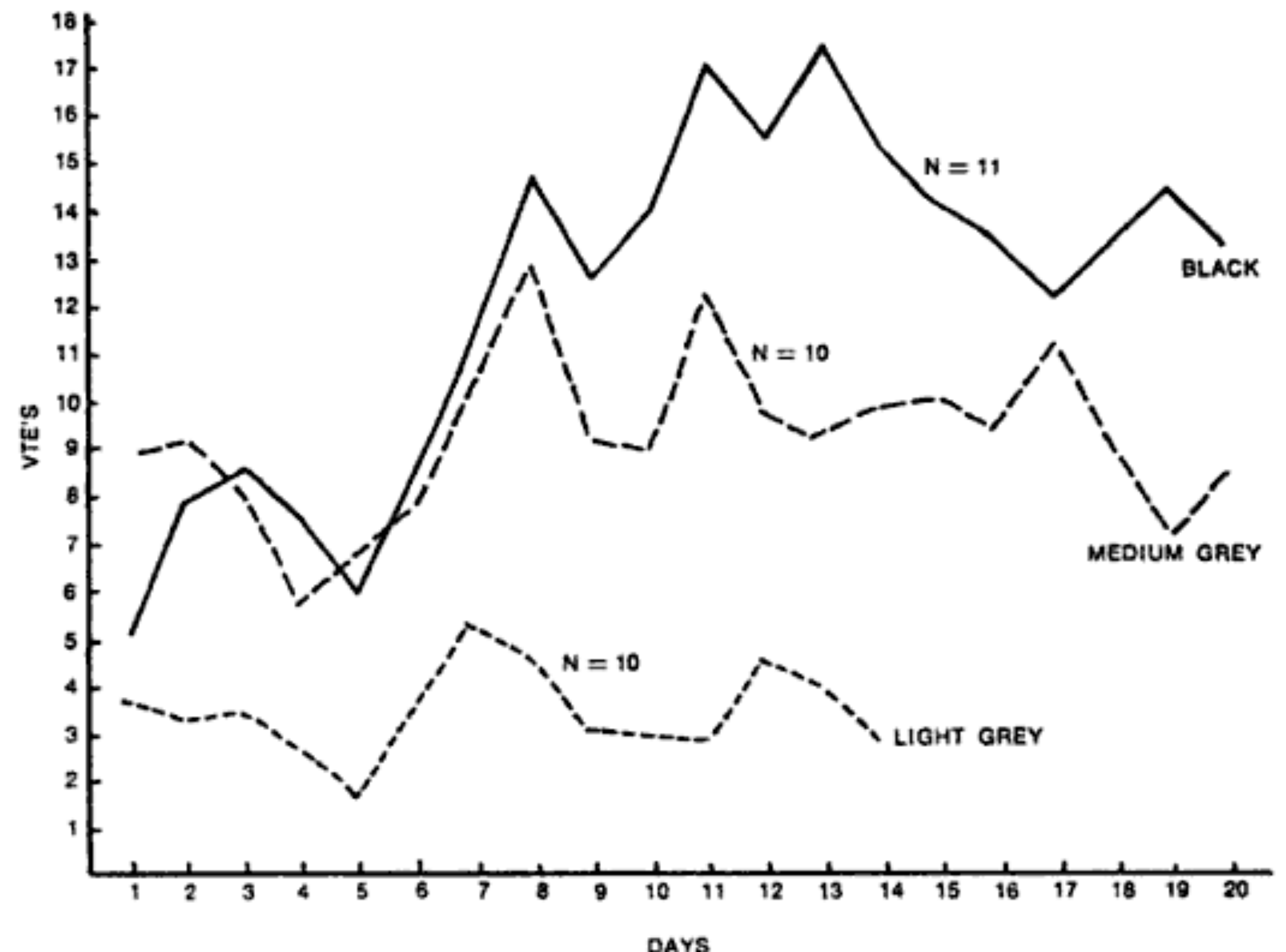
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- VTEs coincide with the start of learning, and fade away afterwards
- Not just passive association of stimuli, but active selecting and comparison of stimuli

## Learning Curves



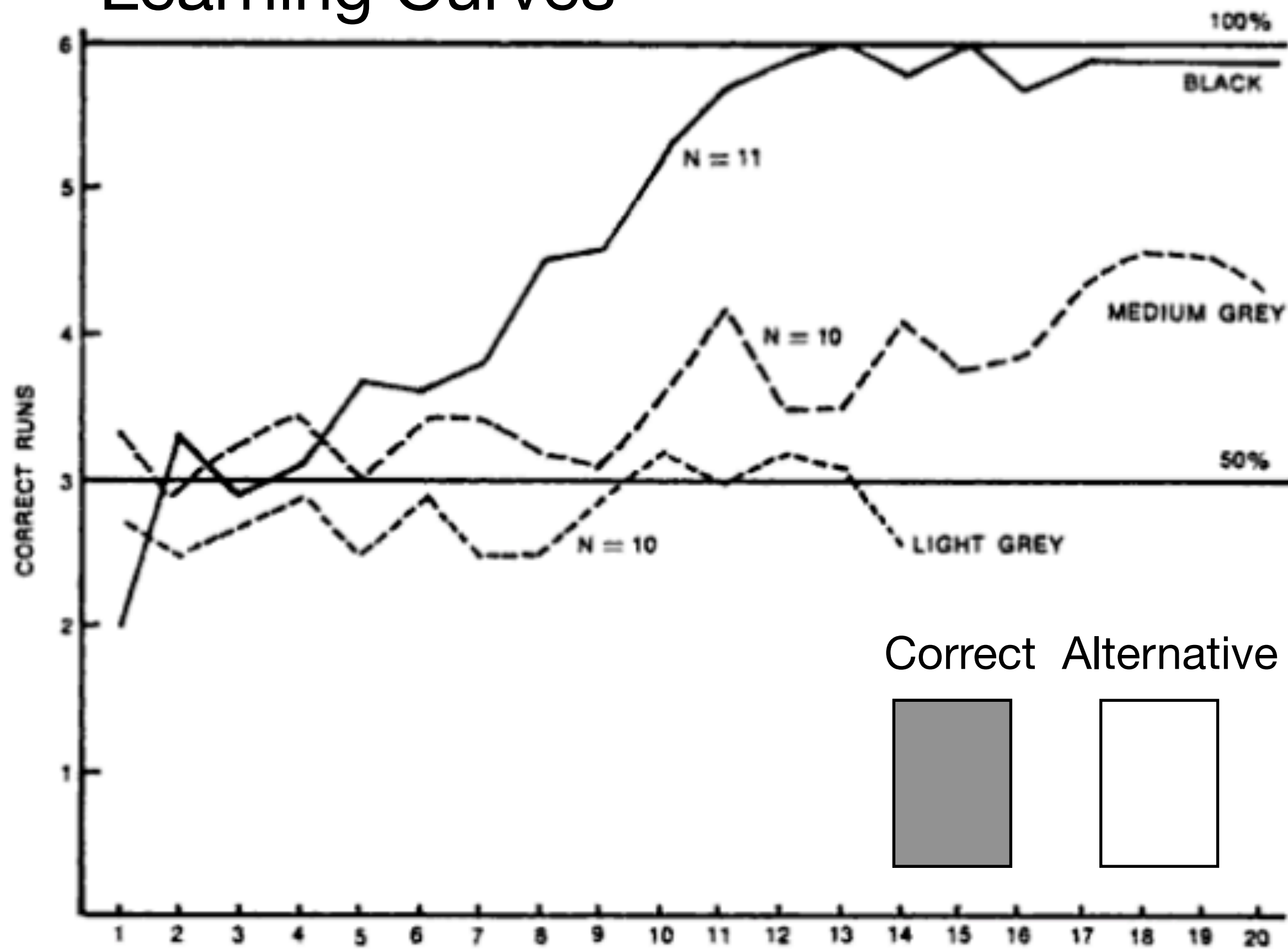
## VTE rate



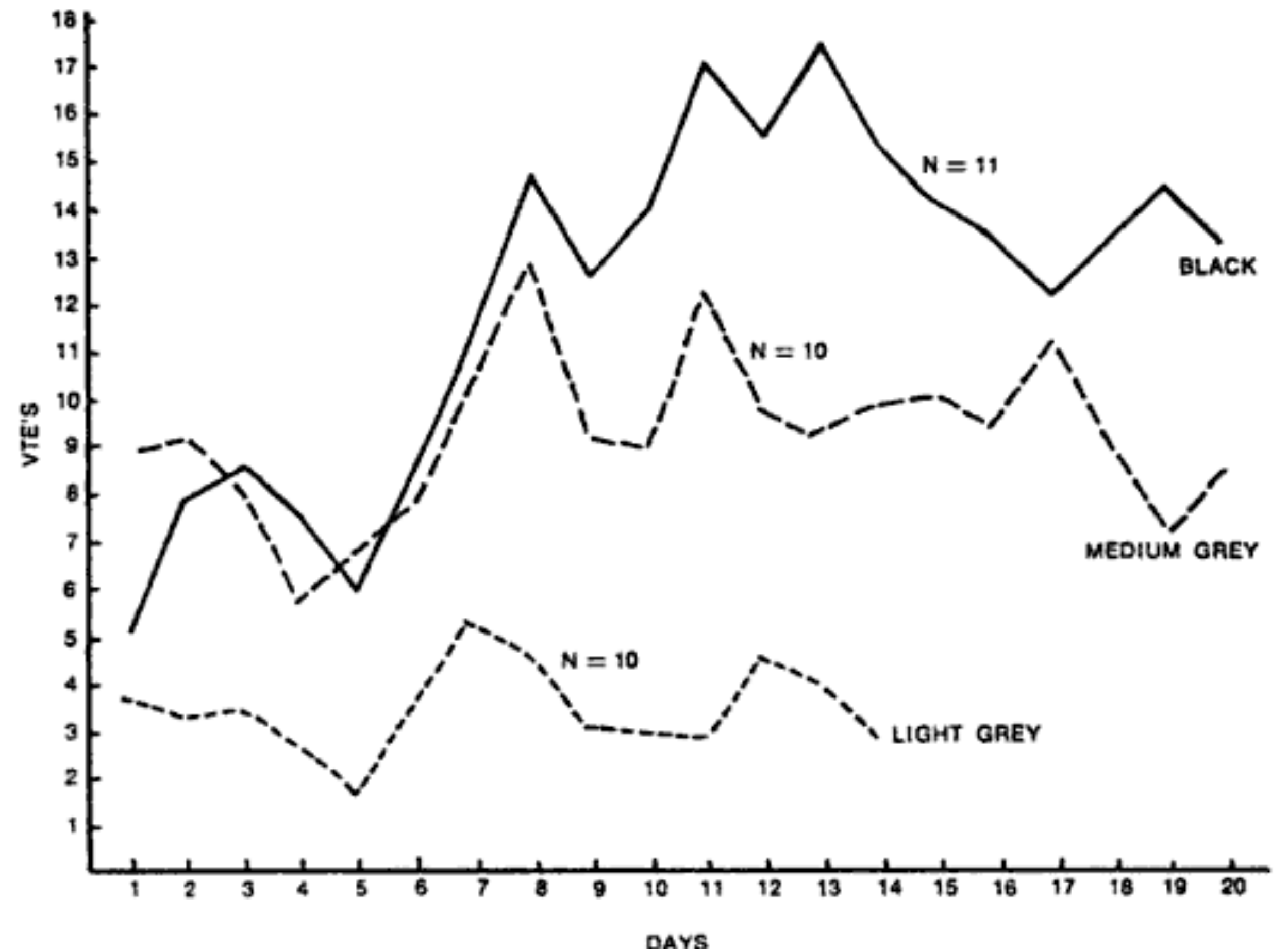
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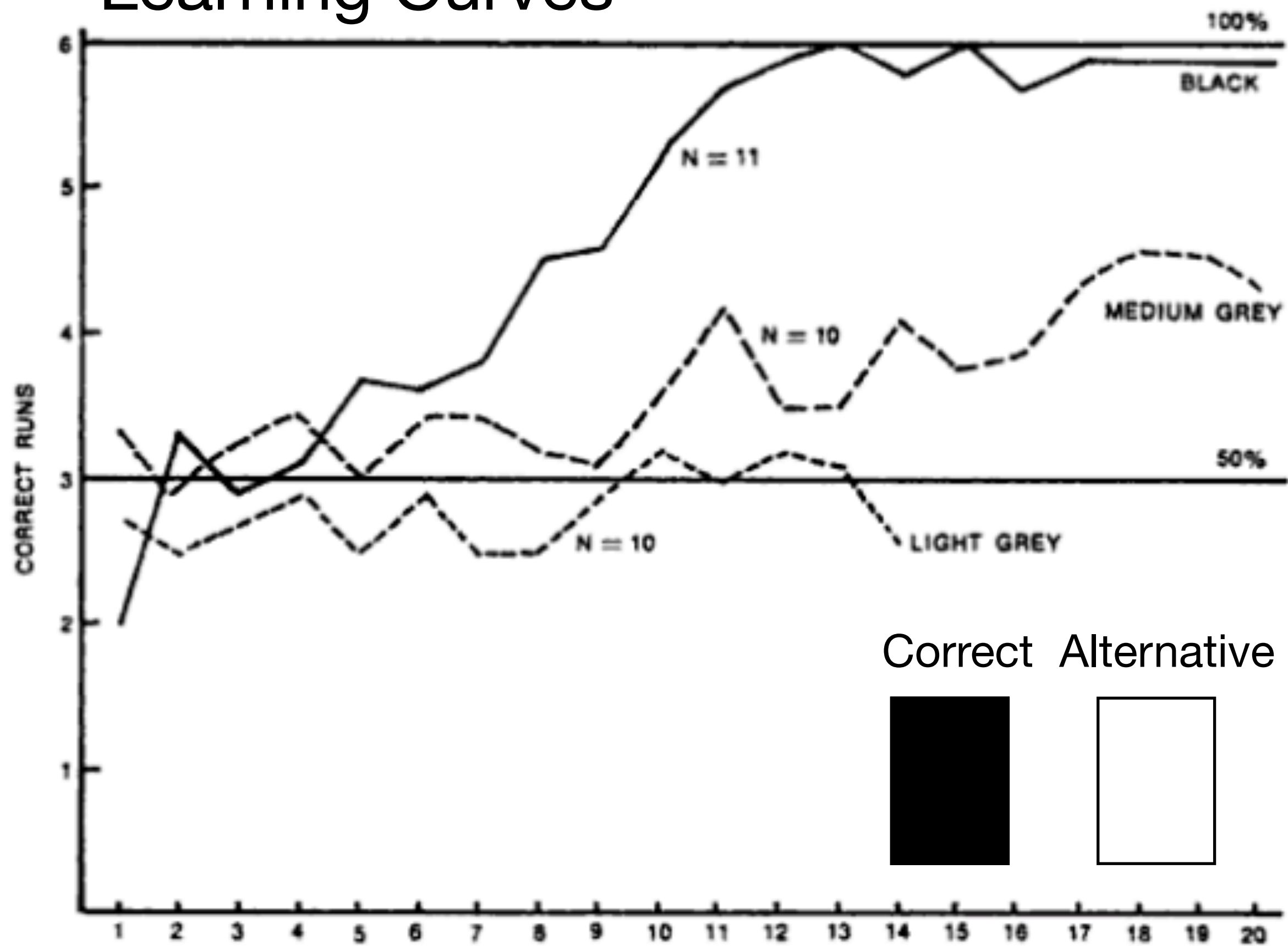
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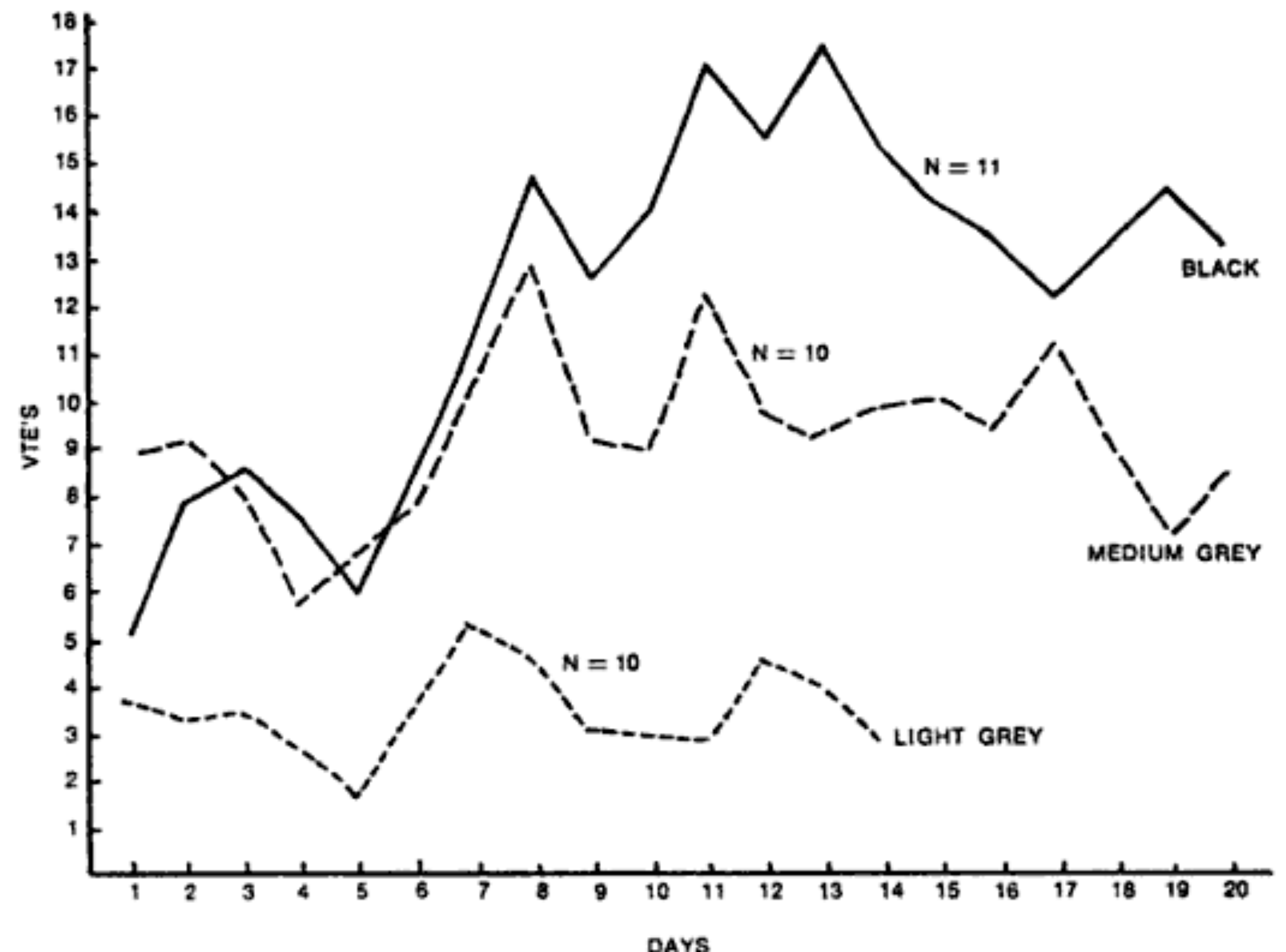
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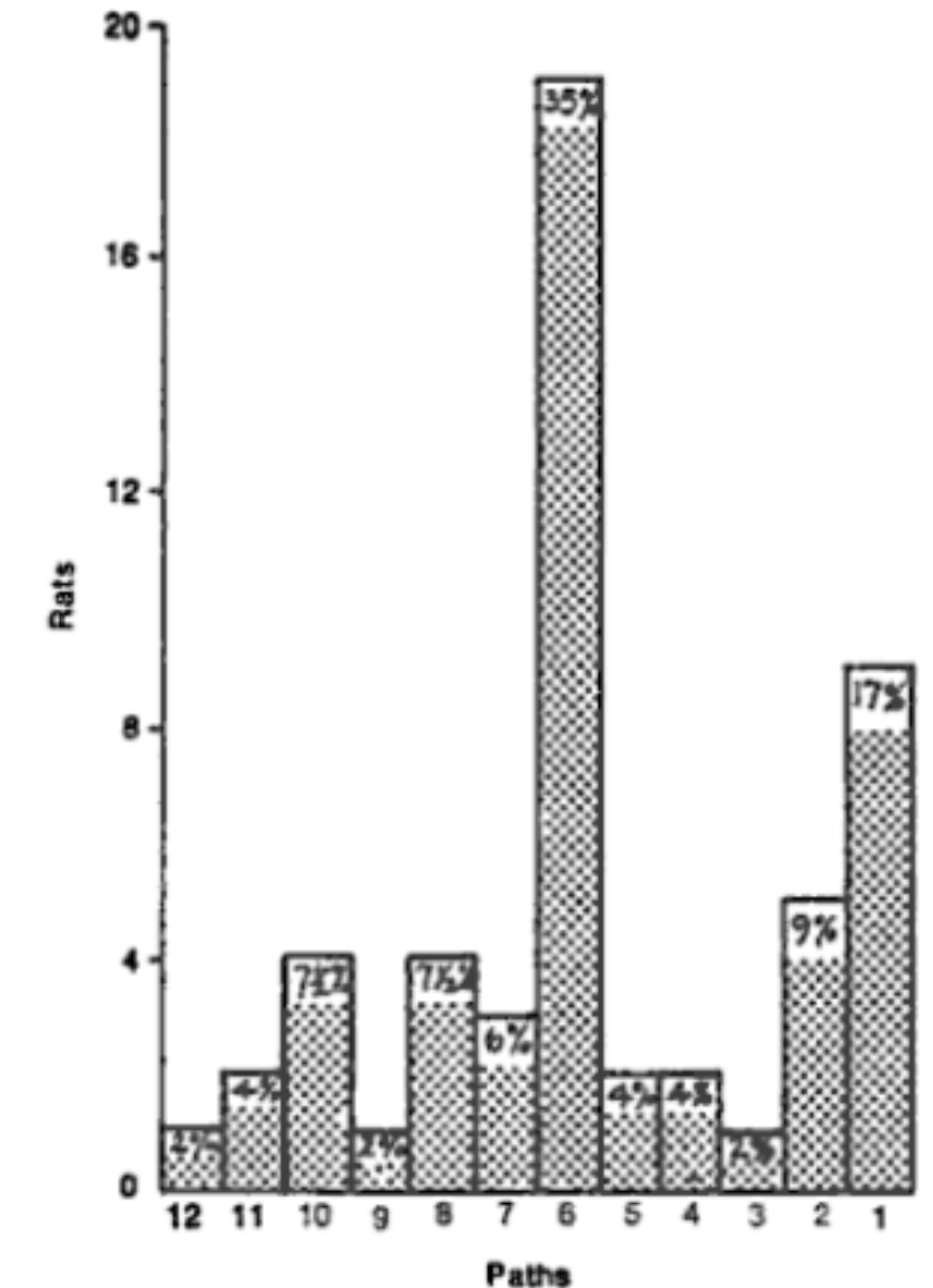
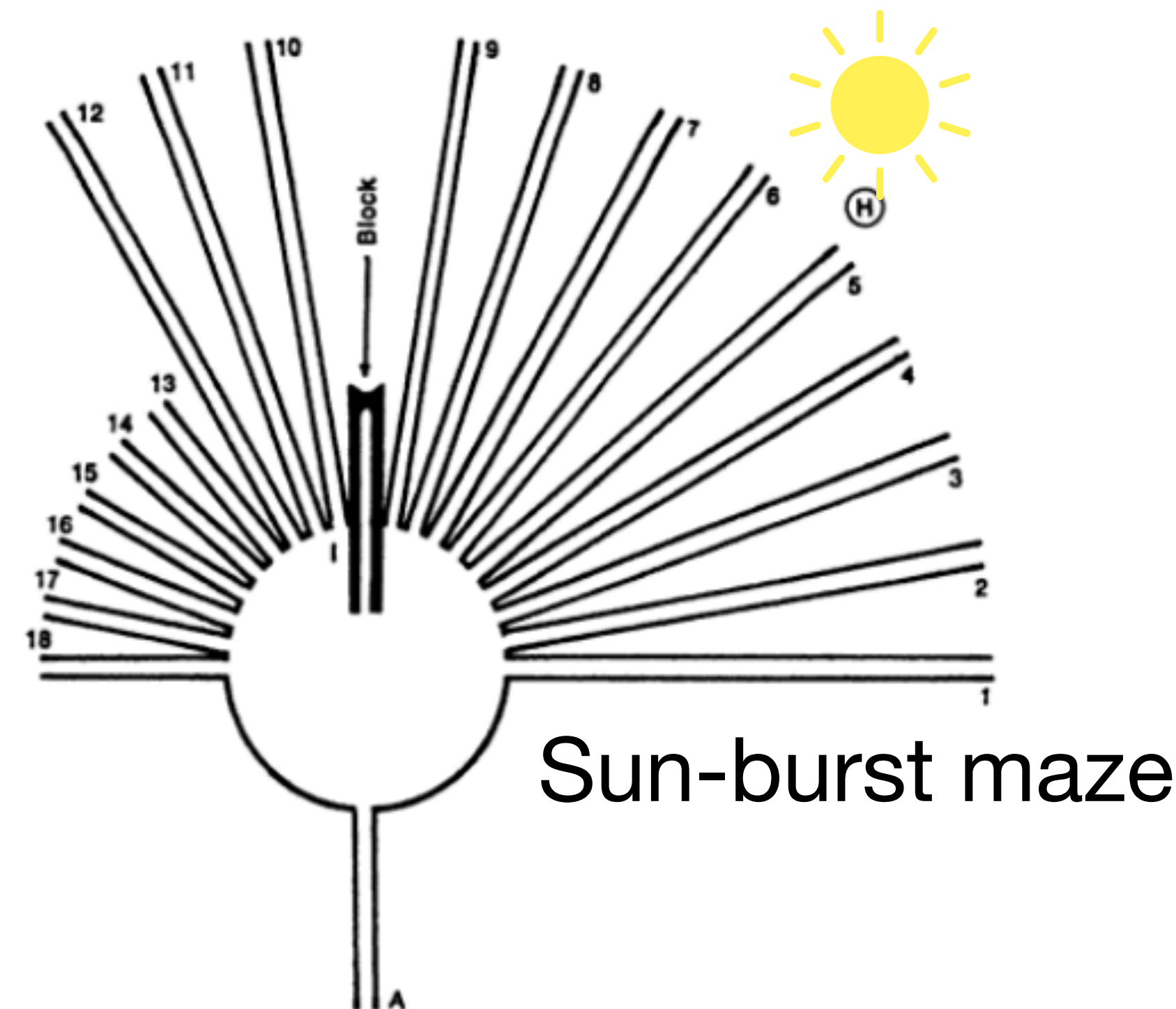
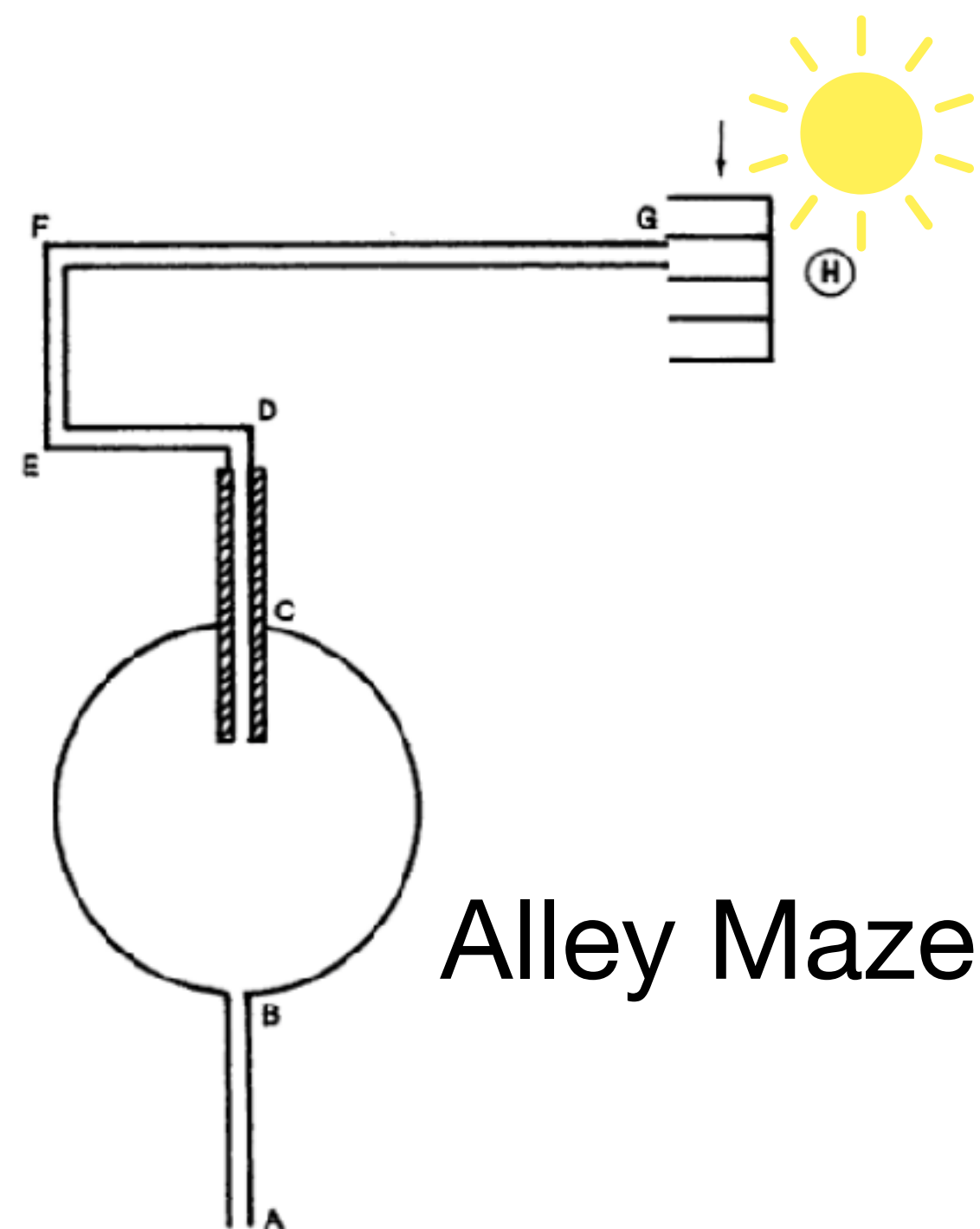
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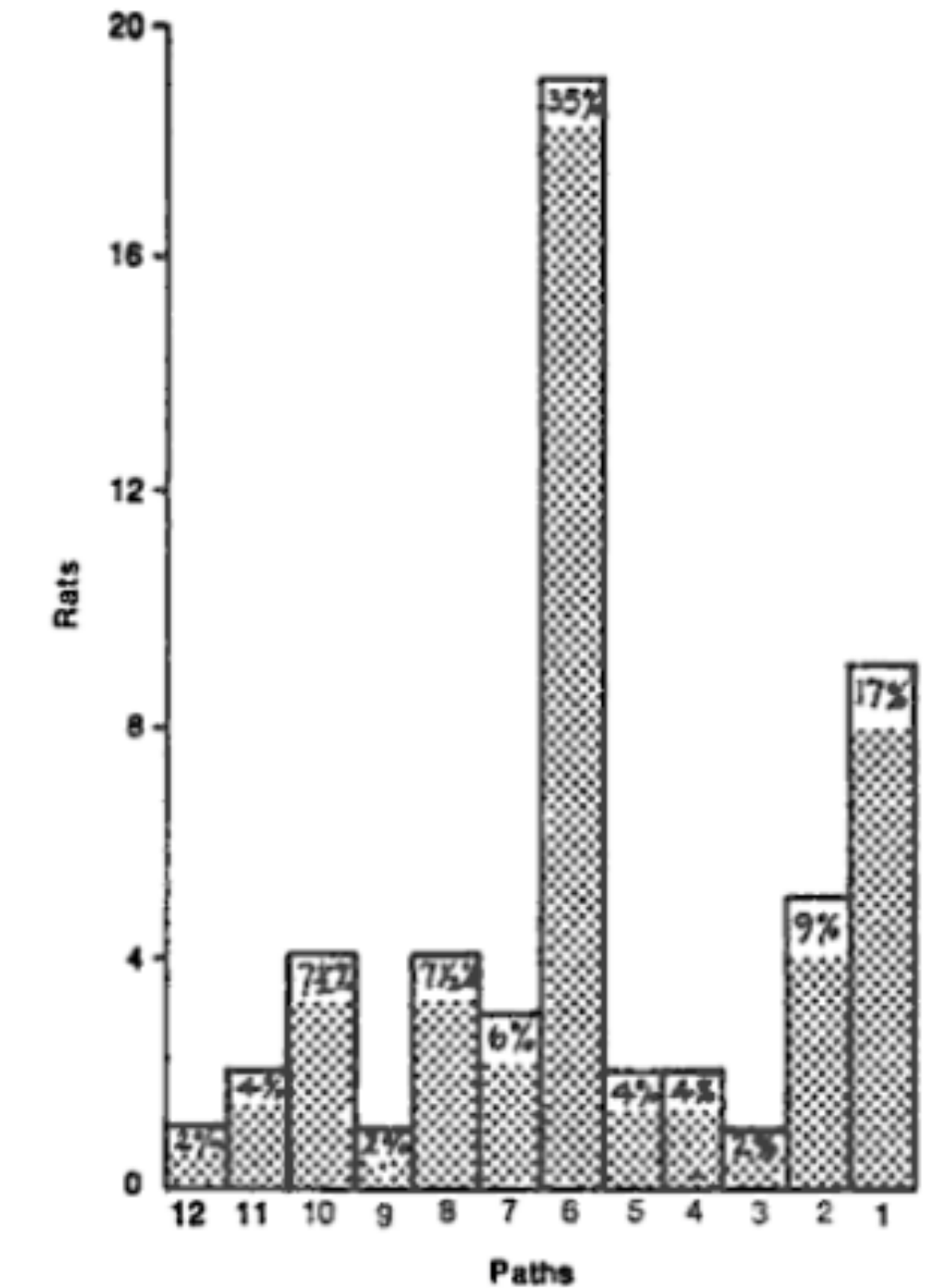
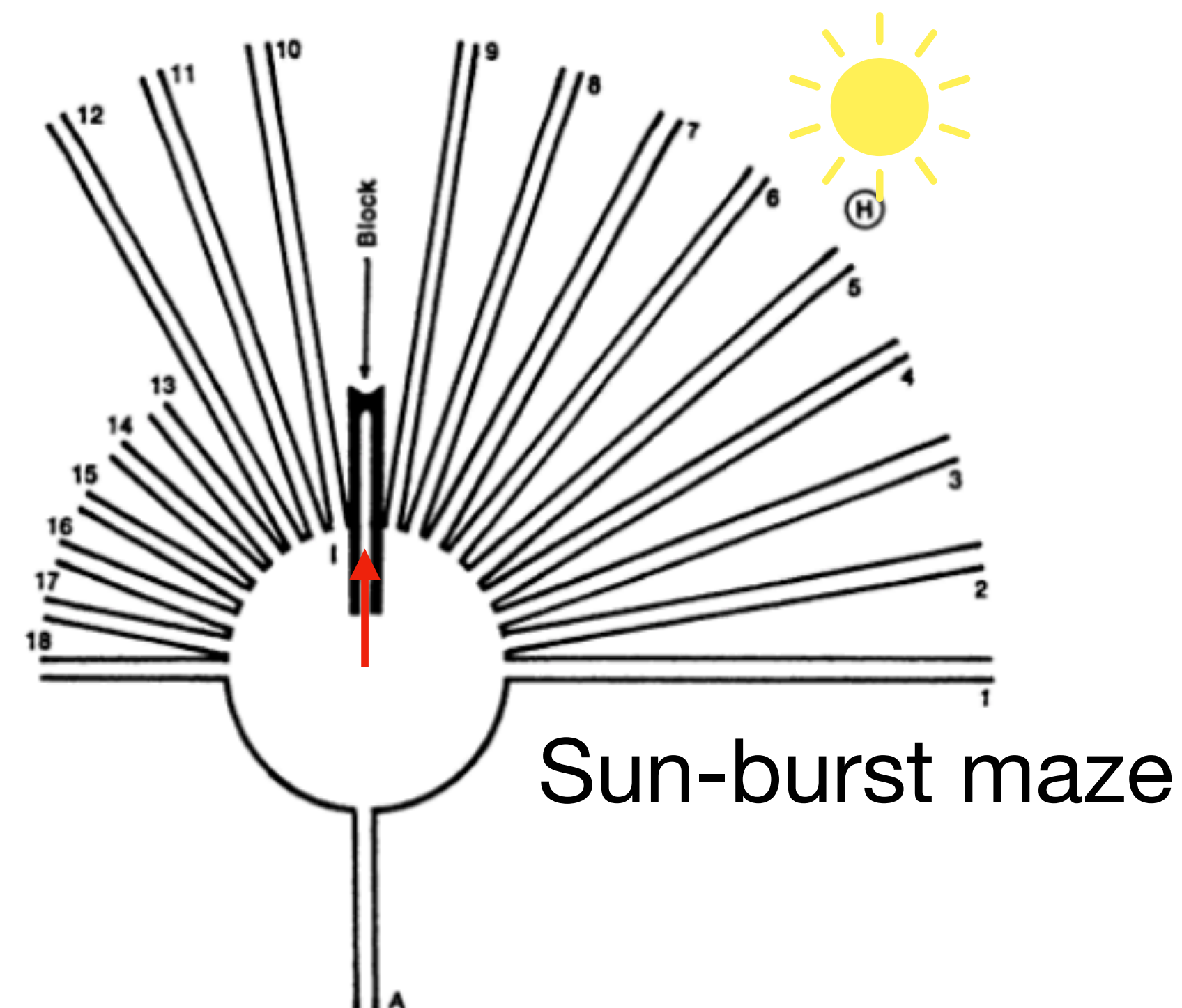
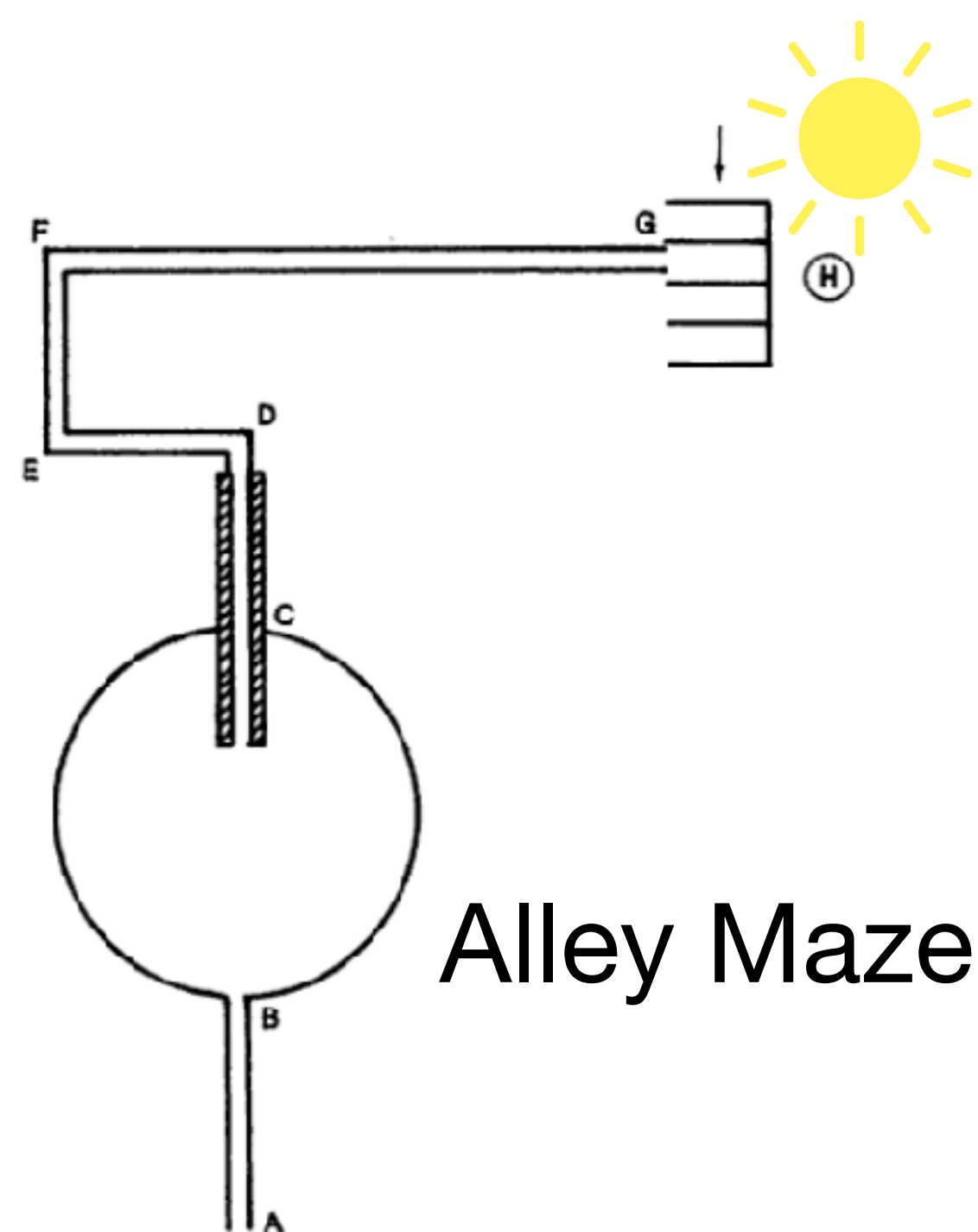
# Spatial Orientation

- 3 trials of alley maze task, where H was a light shining from G-F
- Afterwards, rats transferred to sun-burst maze
  - Initially tried the C-D move, but found it blocked
  - Returned to circle and preferred the radiating path in the same direction as the original food location



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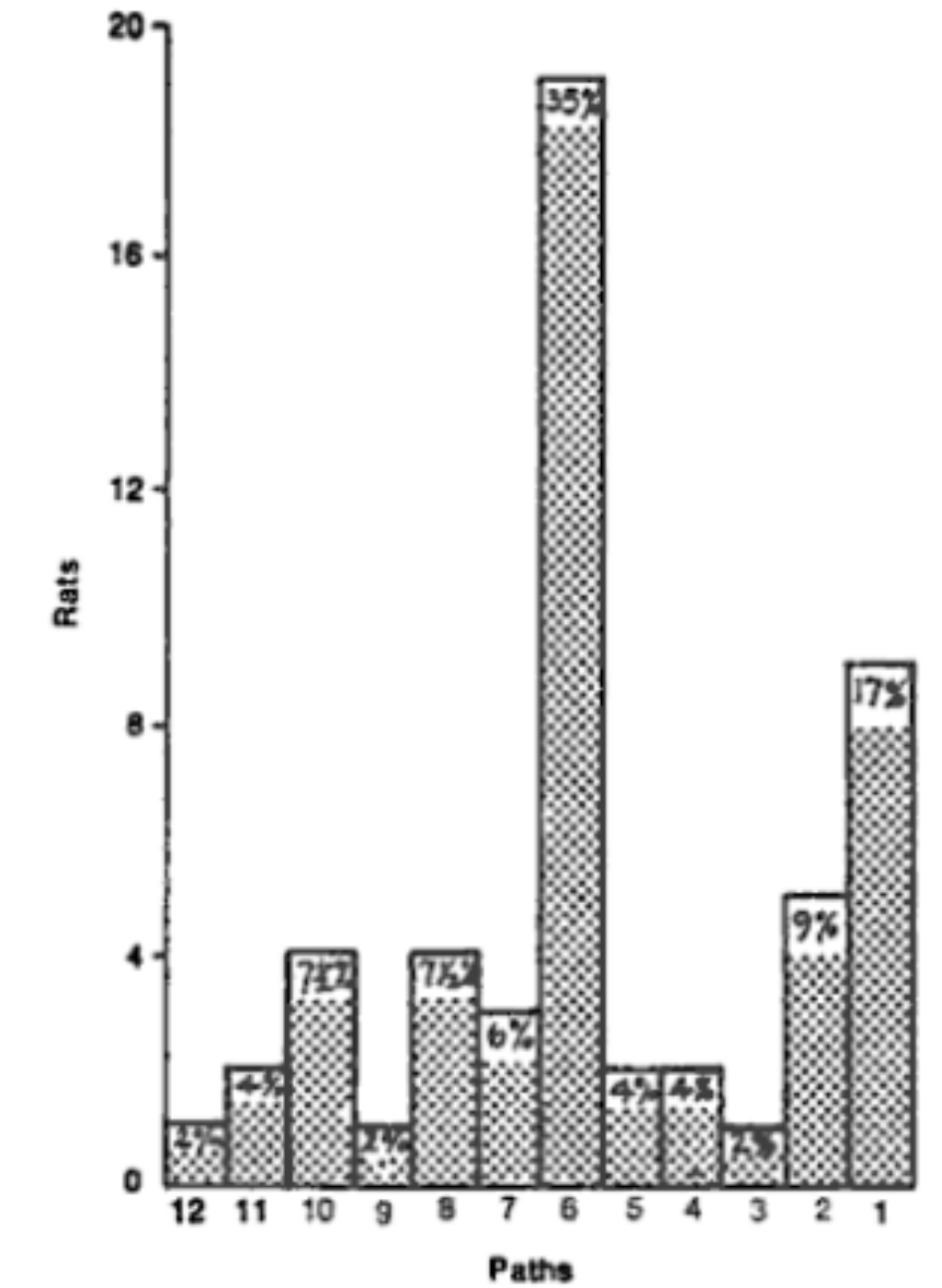
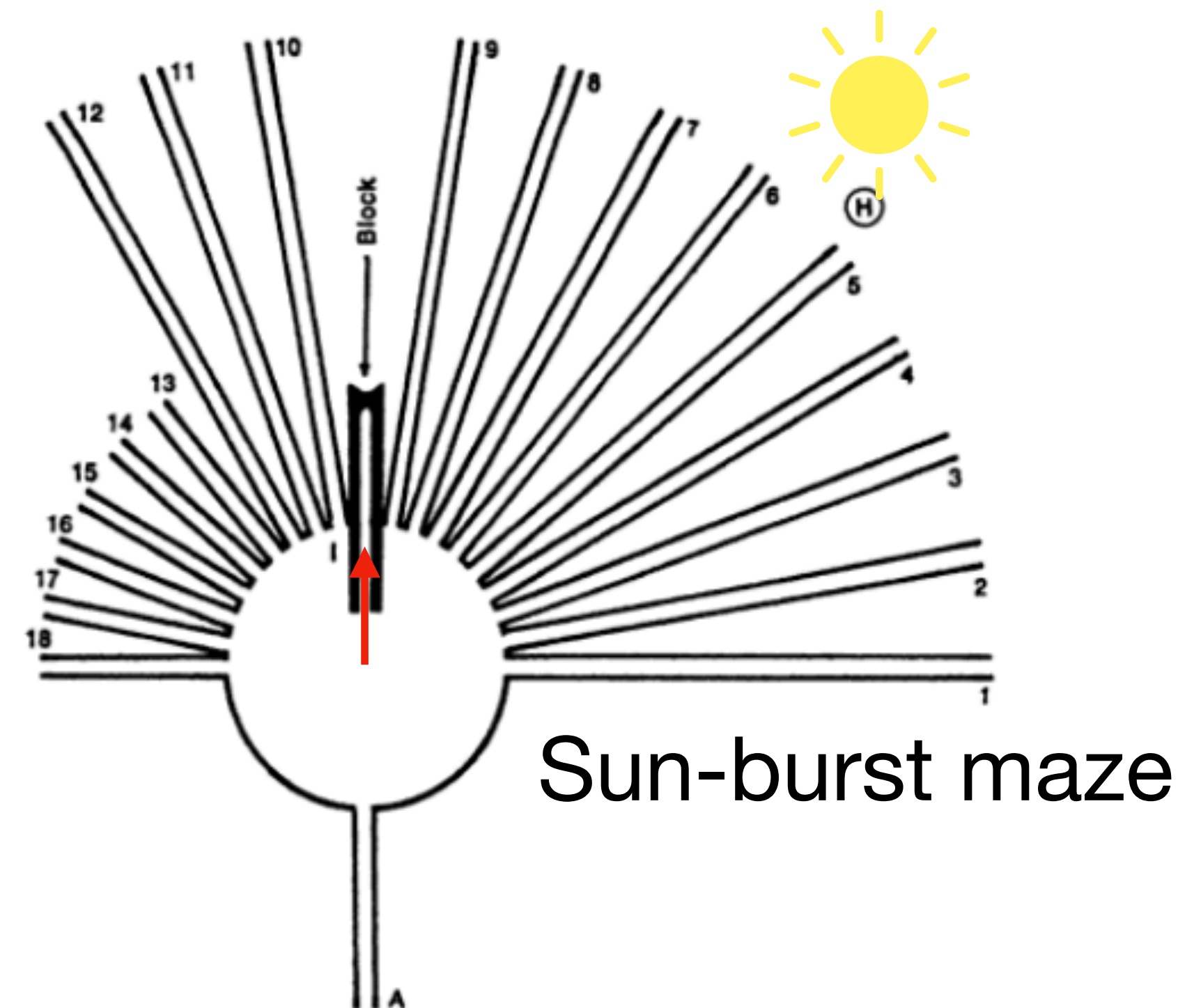
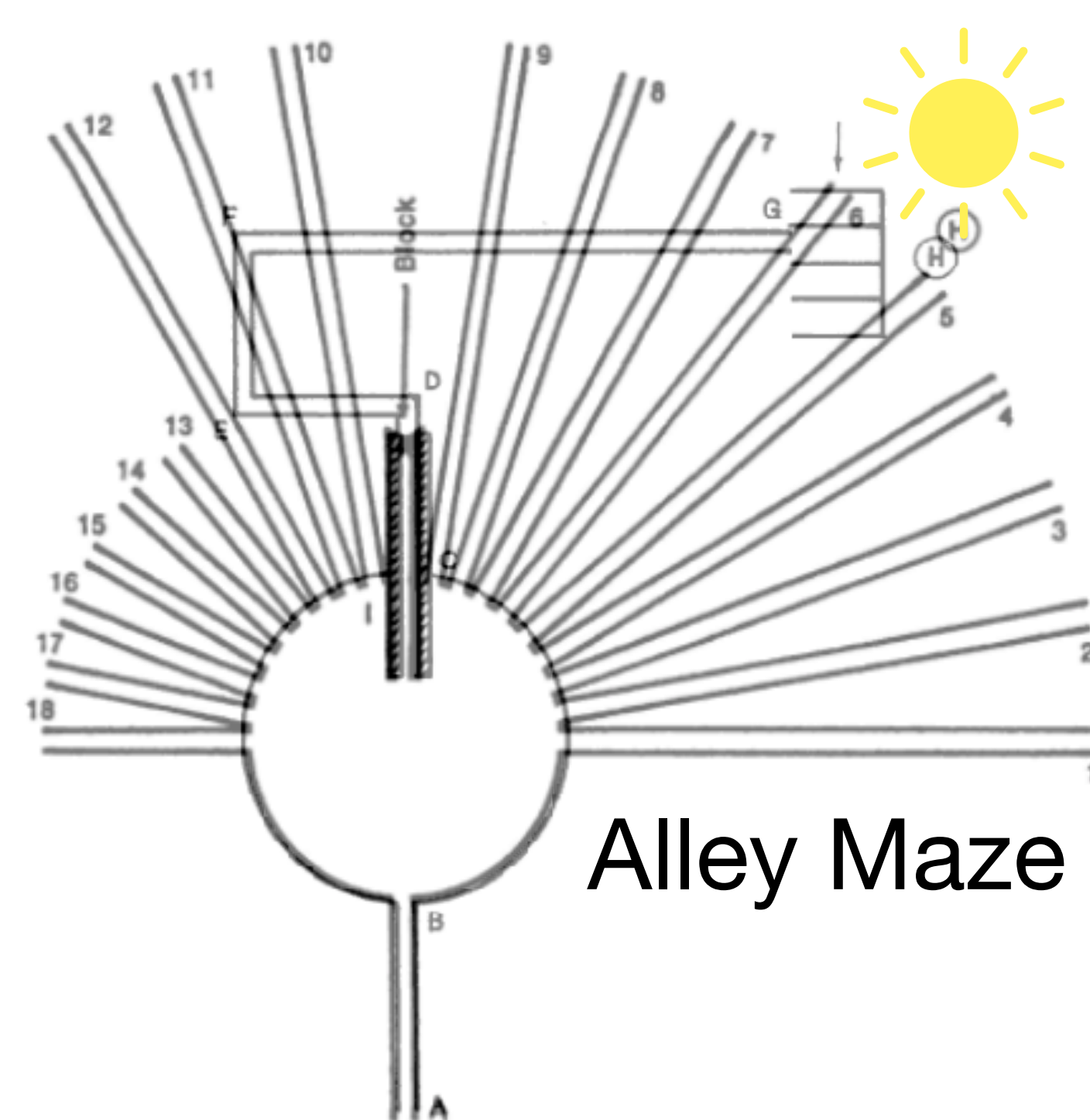
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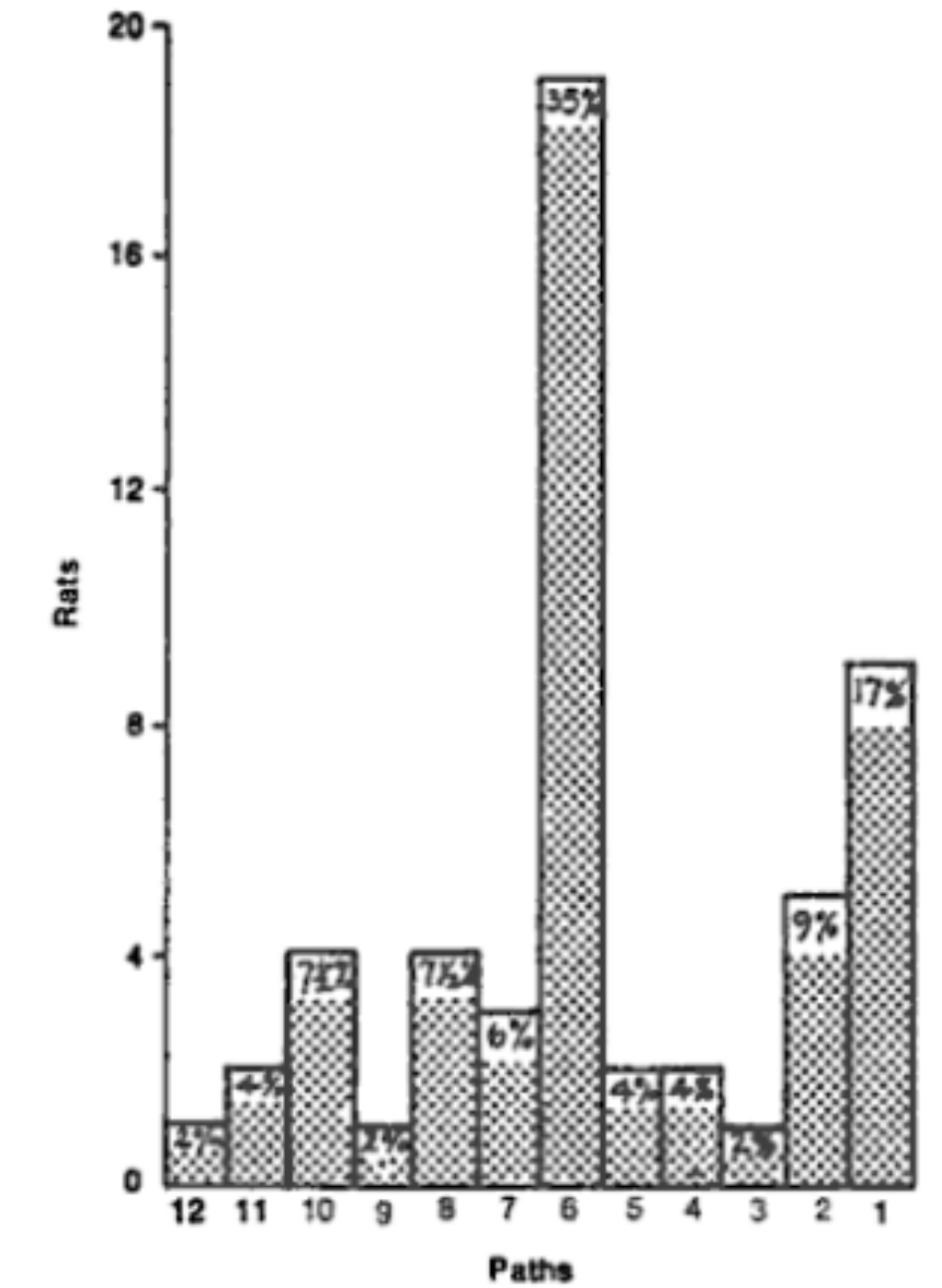
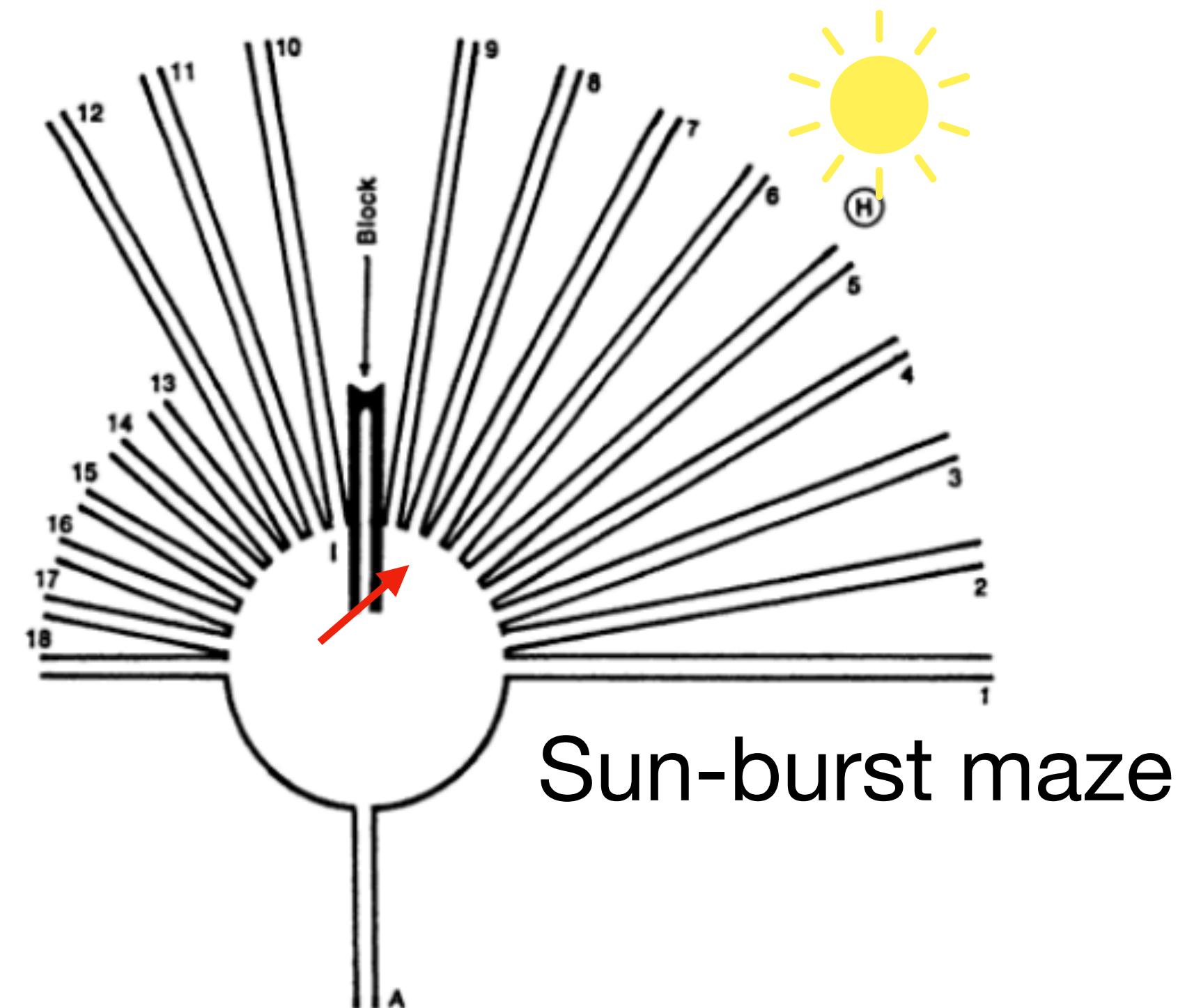
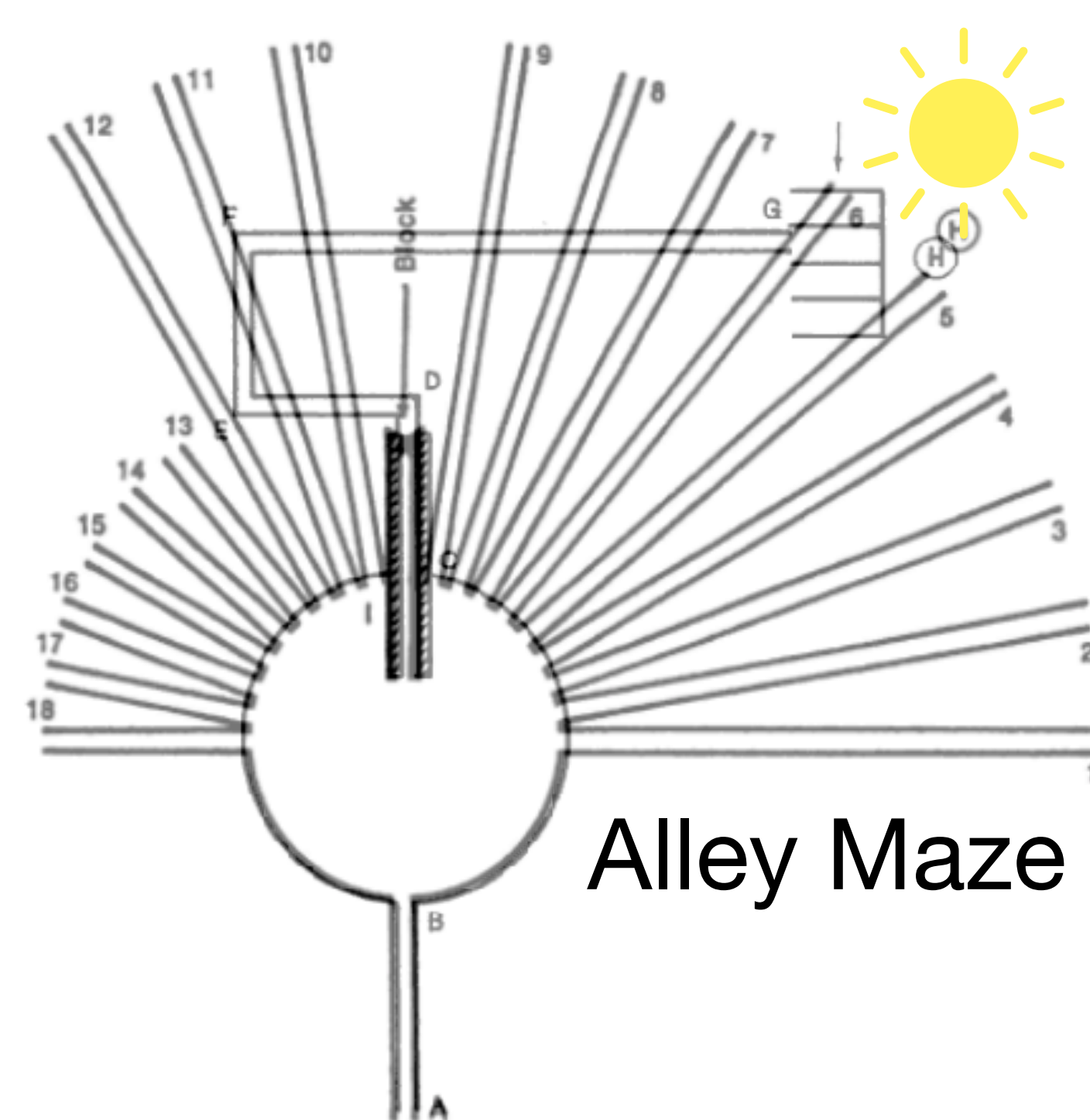
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# Cognitive Maps shape generalization

- The nature of the maps we learn shape how we generalize
  - *“the narrower and more strip-like the original map, the less will it carry over successfully to the new problem; whereas, the wider and the more comprehensive it was, the more adequately it will serve in the new set-up”*
- What conditions favor learning a narrow strip-map vs. a broad comprehensive map?
  - narrow maps induced by :
    - 1) damaged brains
    - 2) impoverished environments
    - 3) overdose of repetition
    - 4) too strongly motivational/frustrating conditions

# Maladaptive psychopathologies

- **Regression** to childlike behavior

*“take an example, the overprotected middle-aged woman [...] who, after losing her husband, regressed [...] into dressing in too youthful a fashion and into competing for their beaux and then finally into behaving like a child requiring continuous care [...].”*

- **Fixation** on various addictive behaviors

*“If rats are too strongly motivated in their original learning, they find it very difficult to relearn when the original path is no longer correct”*

- **Displacement** of aggression towards outgroups

- *“The individual comes no longer to distinguish the true locus of the cause of his frustration”*
- *“The poor Southern whites, who take it out on the Negroes, are displacing their aggressions from the landlords”*
- *“the southern economic system, the northern capitalists, or wherever the true cause of their frustration may lie, [displace their frustration] onto a mere convenient outgroup*
- *[physicists vs. humanities, psychologists vs. all other depts., university vs. secondary school, americans vs. russians]...*
- *“nothing more than such irrational displacements of our aggressions onto outgroups”*

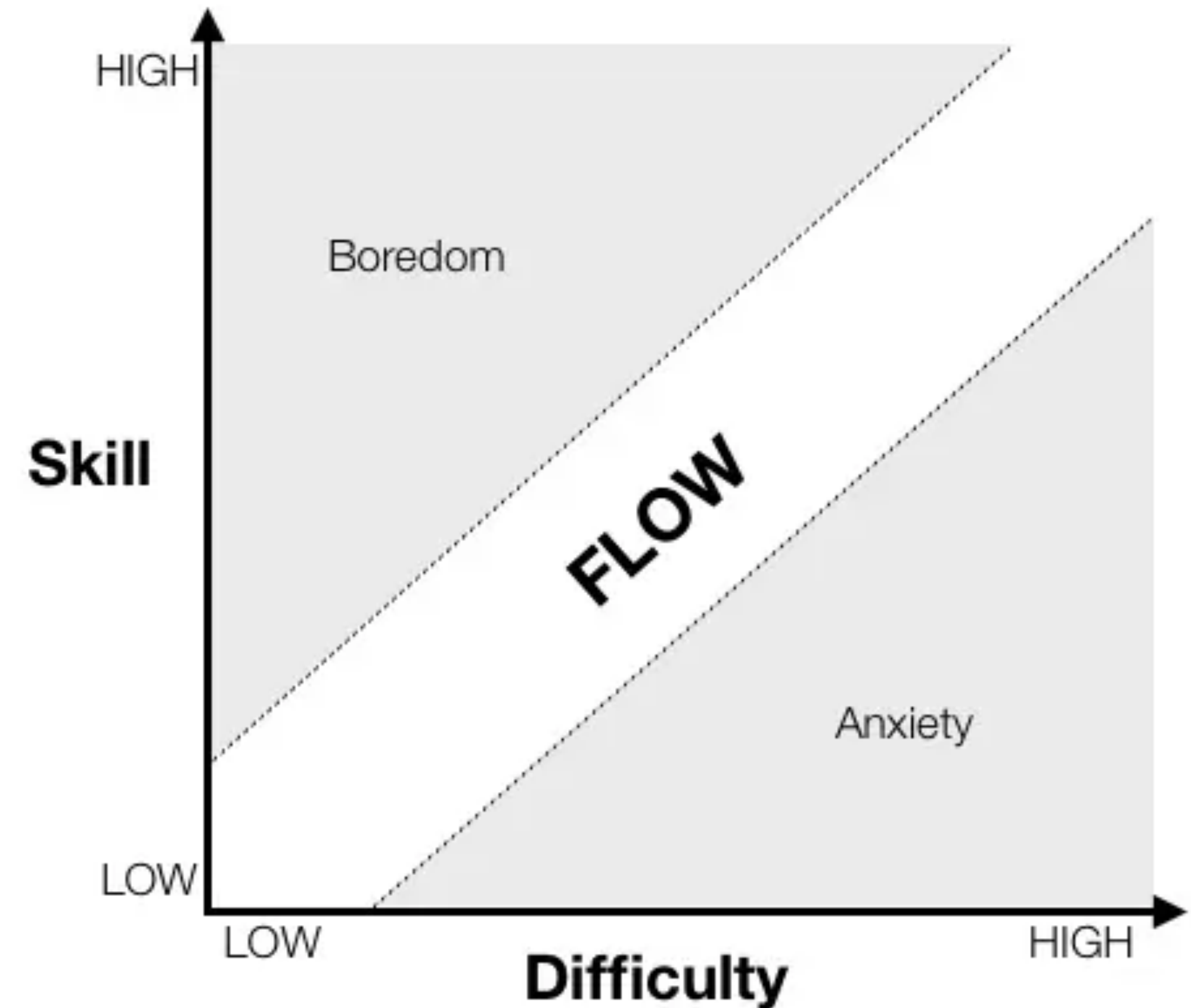


# What is the solution?

*“We must, in short, subject our children and ourselves ... to the optimal conditions of moderate motivation and of an absence of unnecessary frustrations.... I cannot predict whether or not we will be able, or be allowed, to do this; but I **can** say that, only insofar as we **are** able and **are** allowed, have we cause for hope.*”

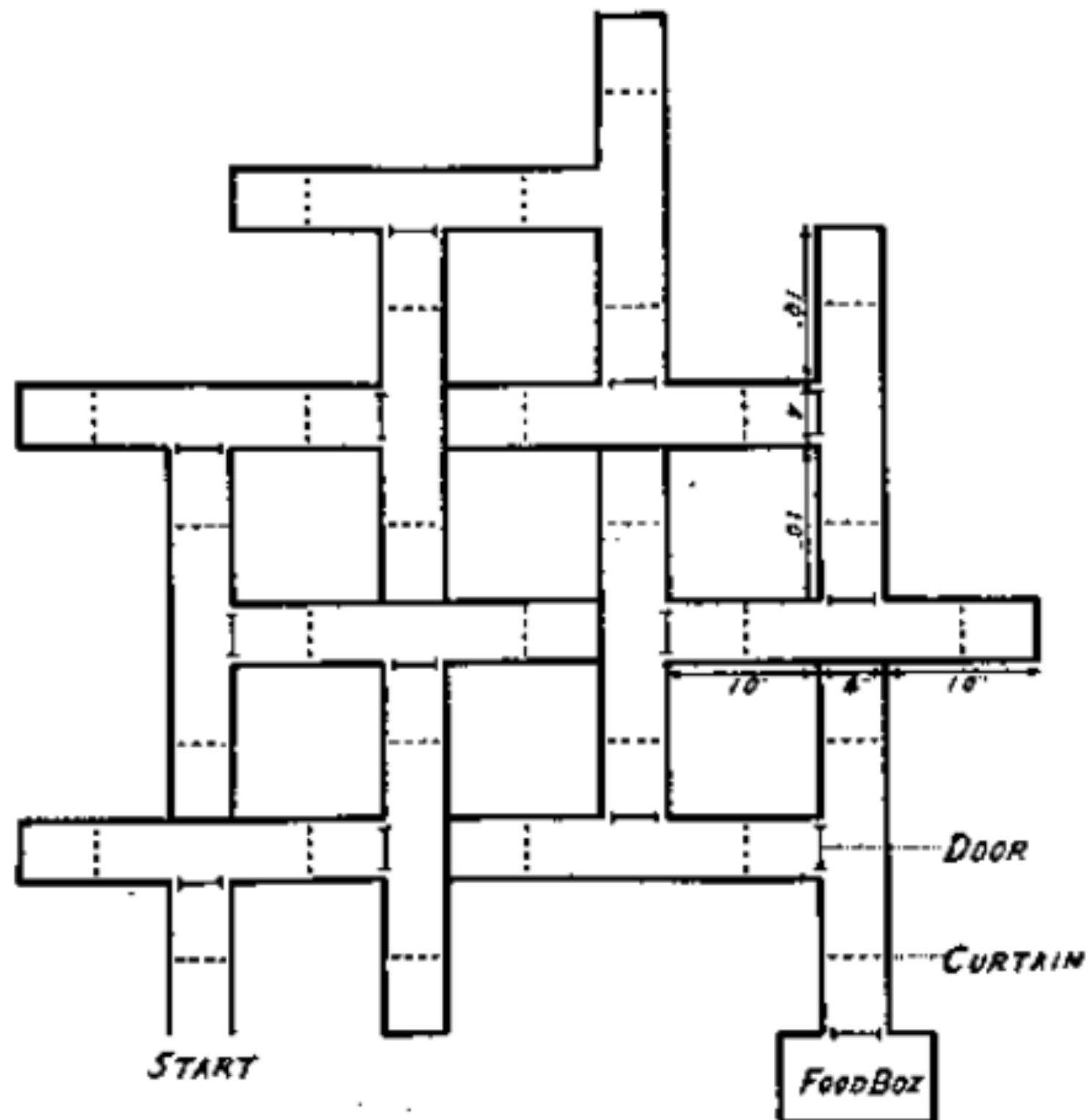
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Csikszentmihalyi (1990)

# Cognitive Maps in the Brain





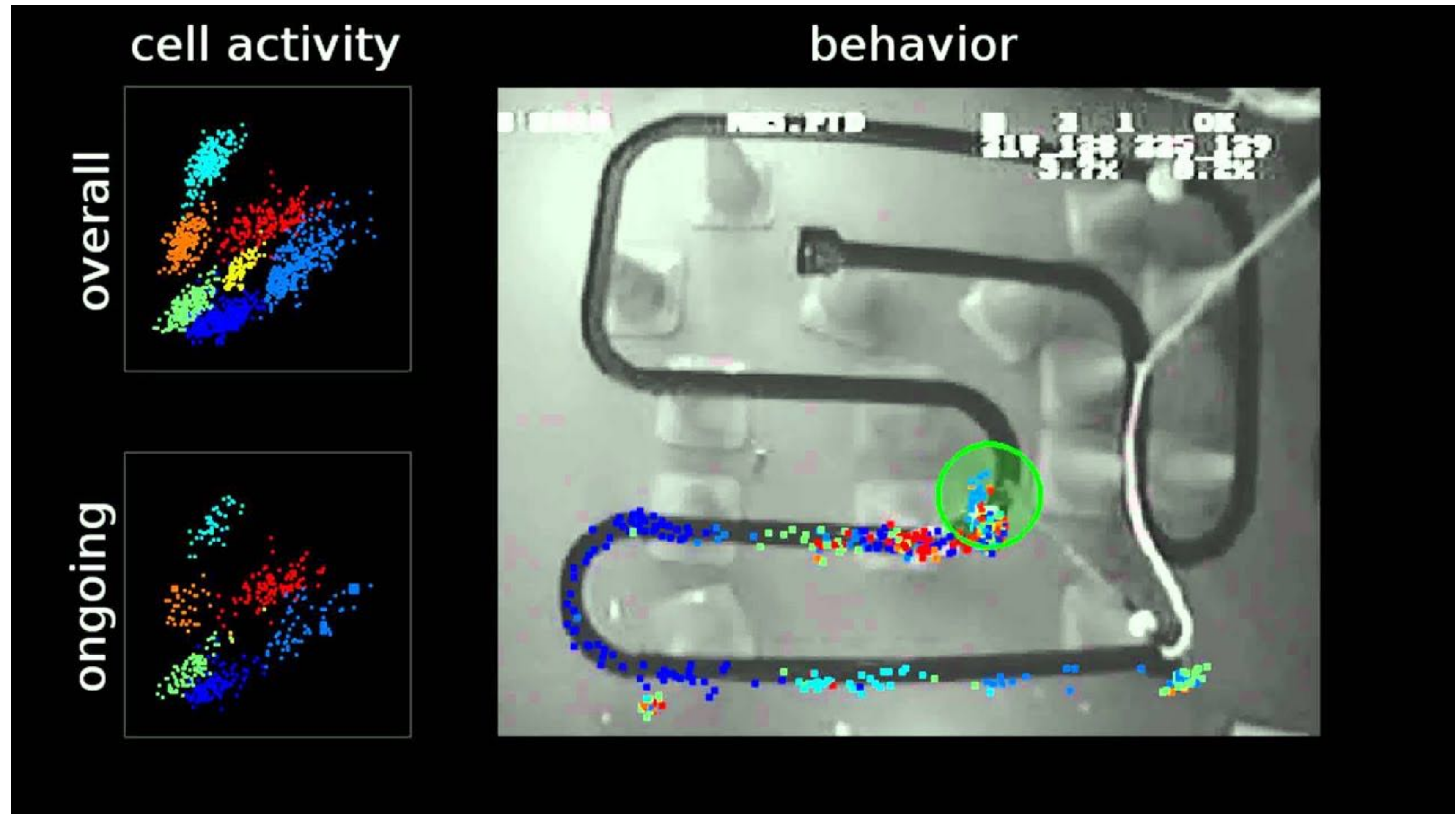
# Place cells in the **hippocampus** represent location in an environment



Place Cell



(O'Keefe & Nadel 1978)



John O'Keefe  
Nobel Prize in Physiology or Medicine 2014

Wilson Lab (MIT)



# Place cells in the **hippocampus** represent location in an environment



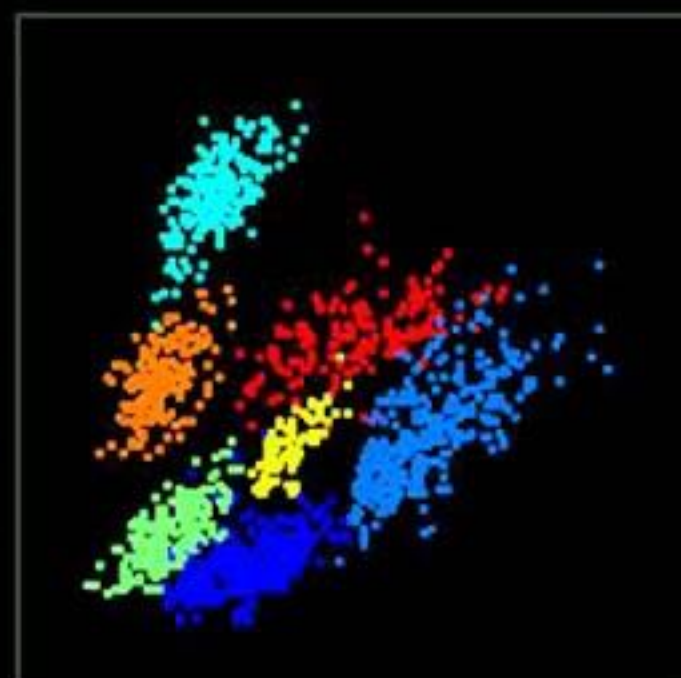
Place Cell



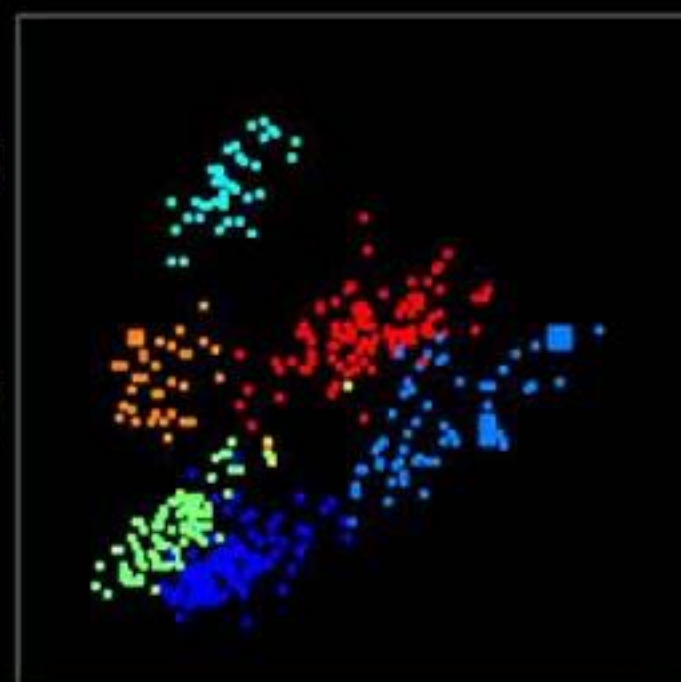
*(O'Keefe & Nadel 1978)*

cell activity

overall



ongoing



behavior

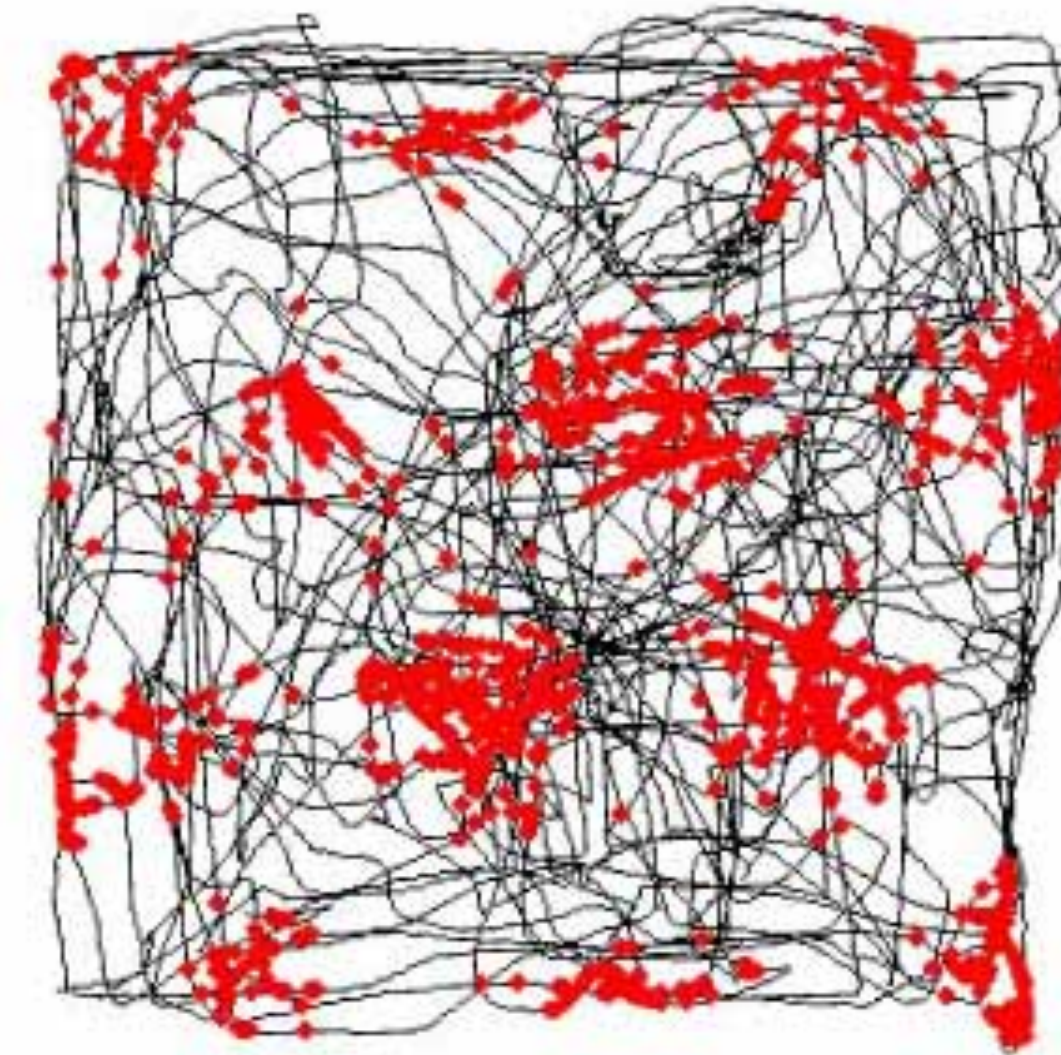
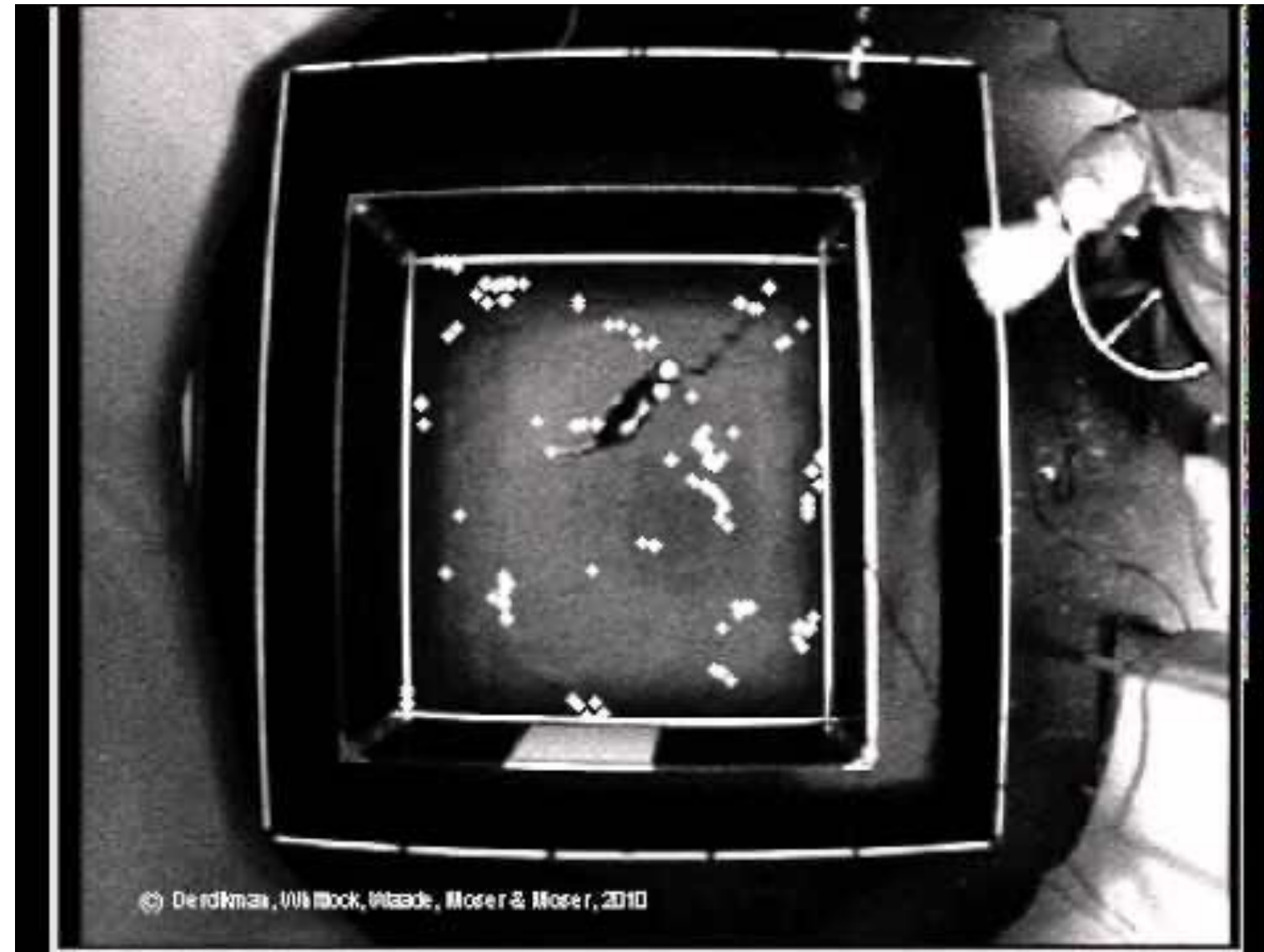


John O'Keefe  
Nobel Prize in Physiology or Medicine 2014

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# Grid cells in the **Entorhinal Cortex** provide a coordinate system

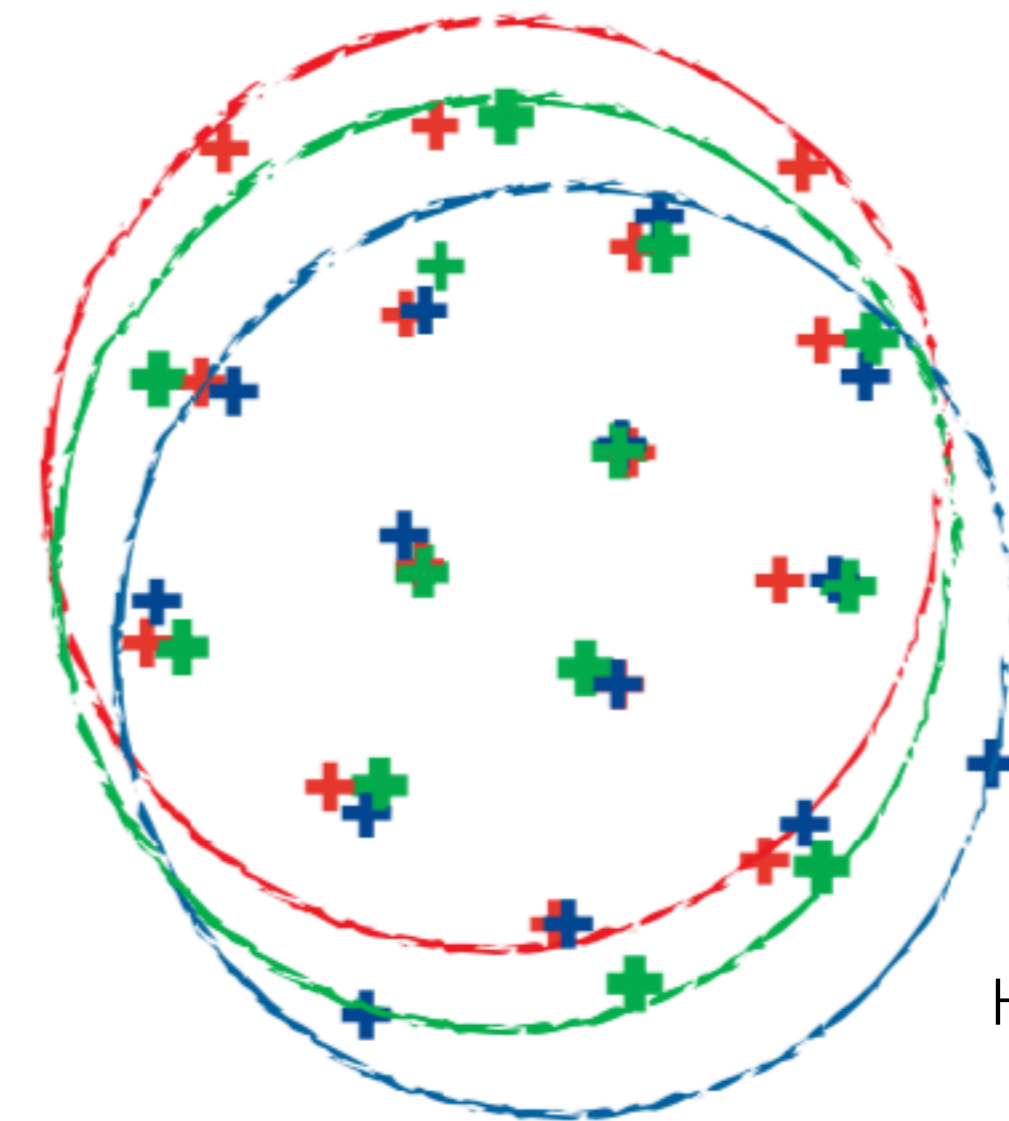
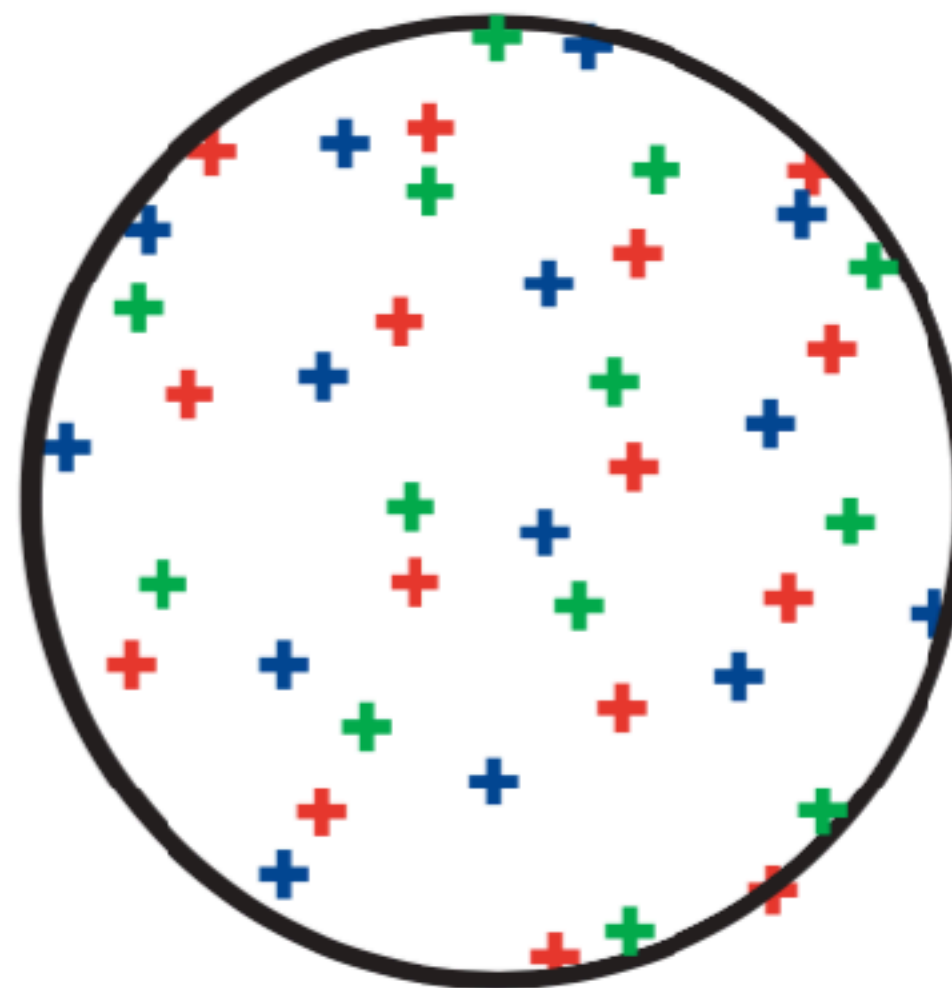
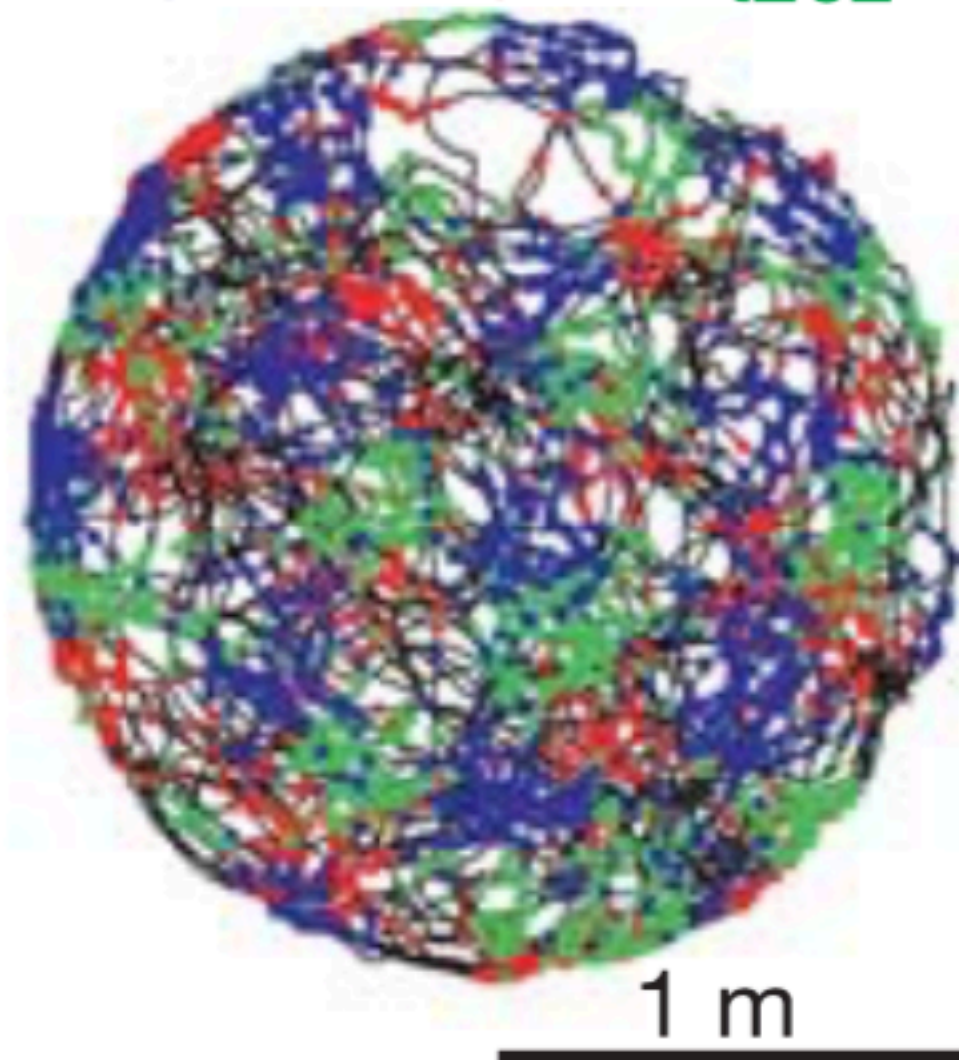


- Trajectory
- Peaks



Edvard and Maj-Britt Moser  
Nobel Prize in Physiology or  
Medicine 2014

t1c1 t2c1 t2c2

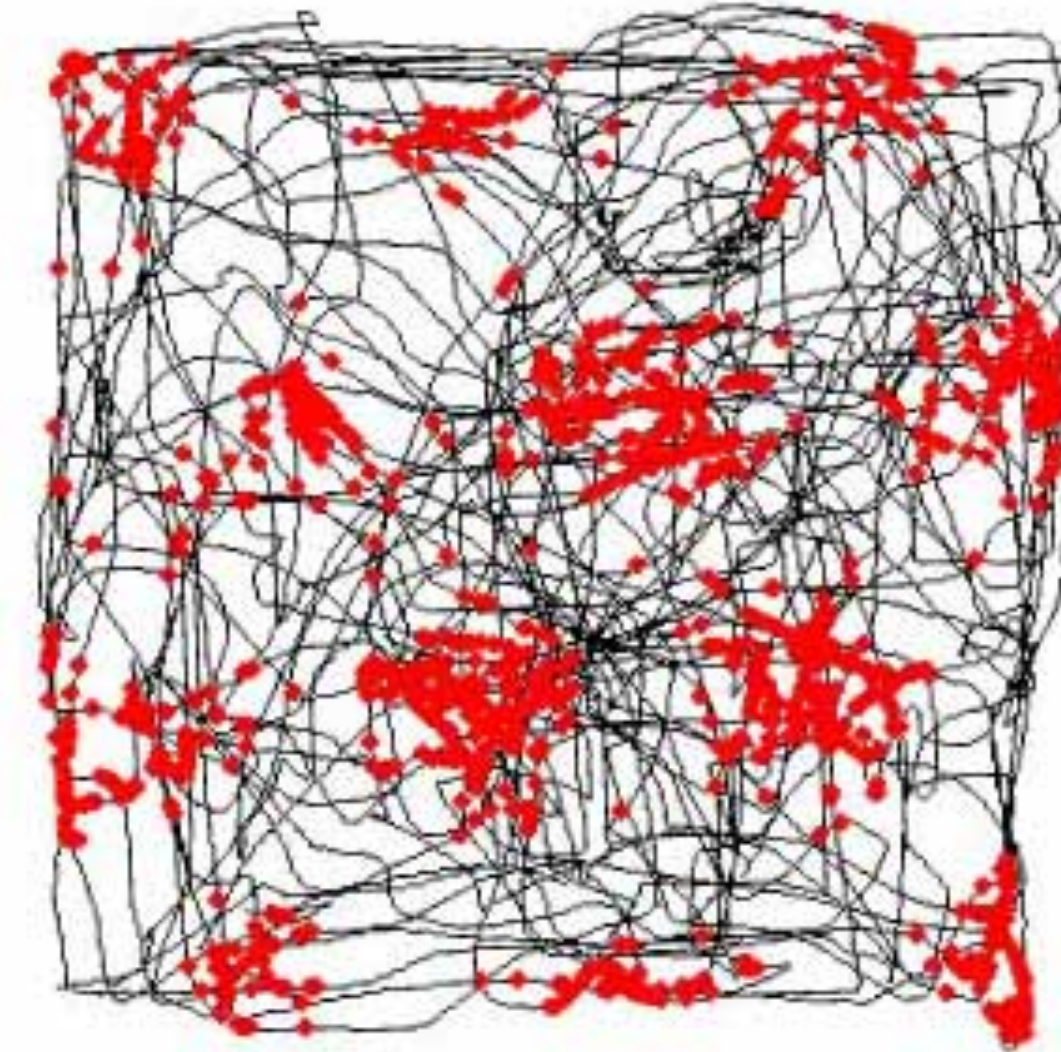
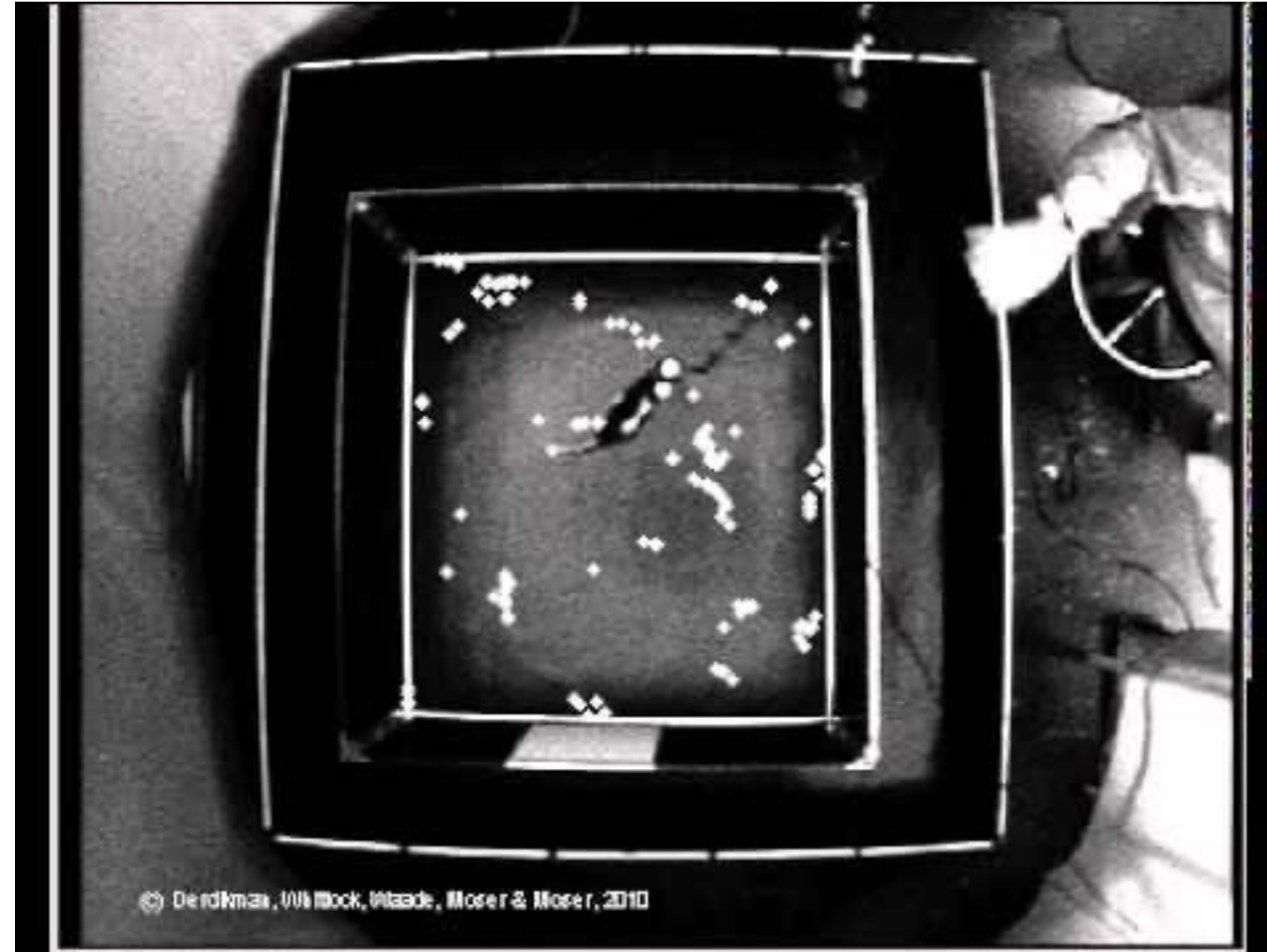


+ Peak

Hafting *et al* (Nature, 2005)



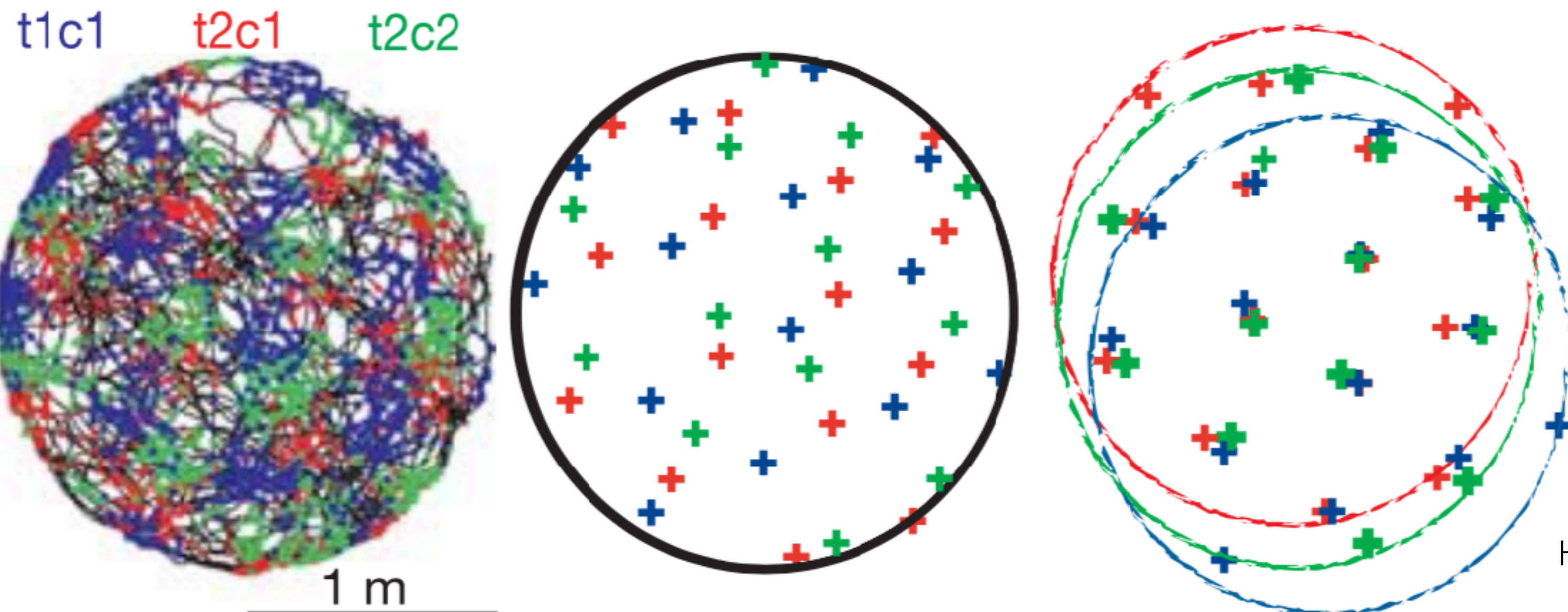
# Grid cells in the **Entorhinal Cortex** provide a coordinate system



- Trajectory
- Peaks



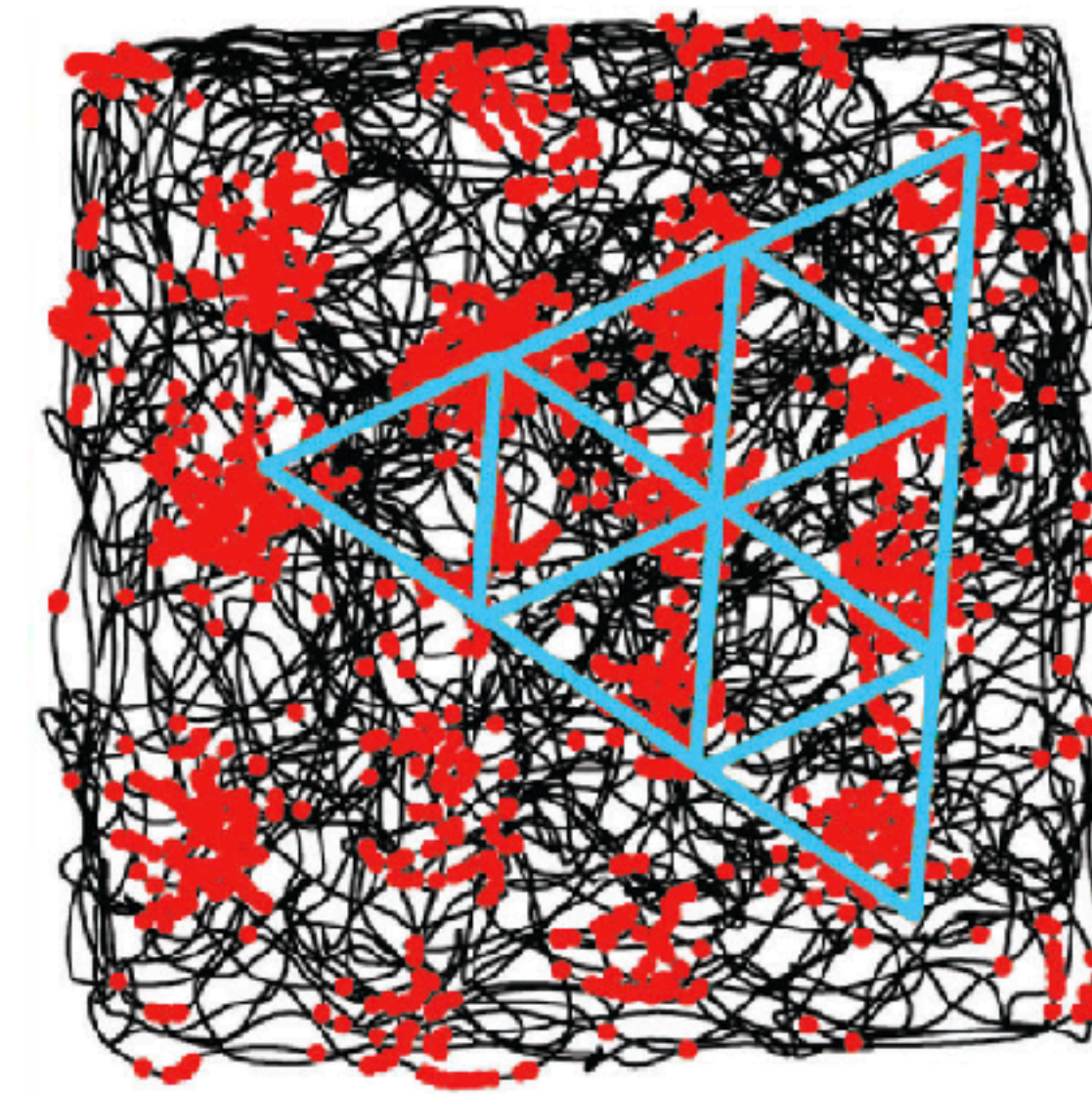
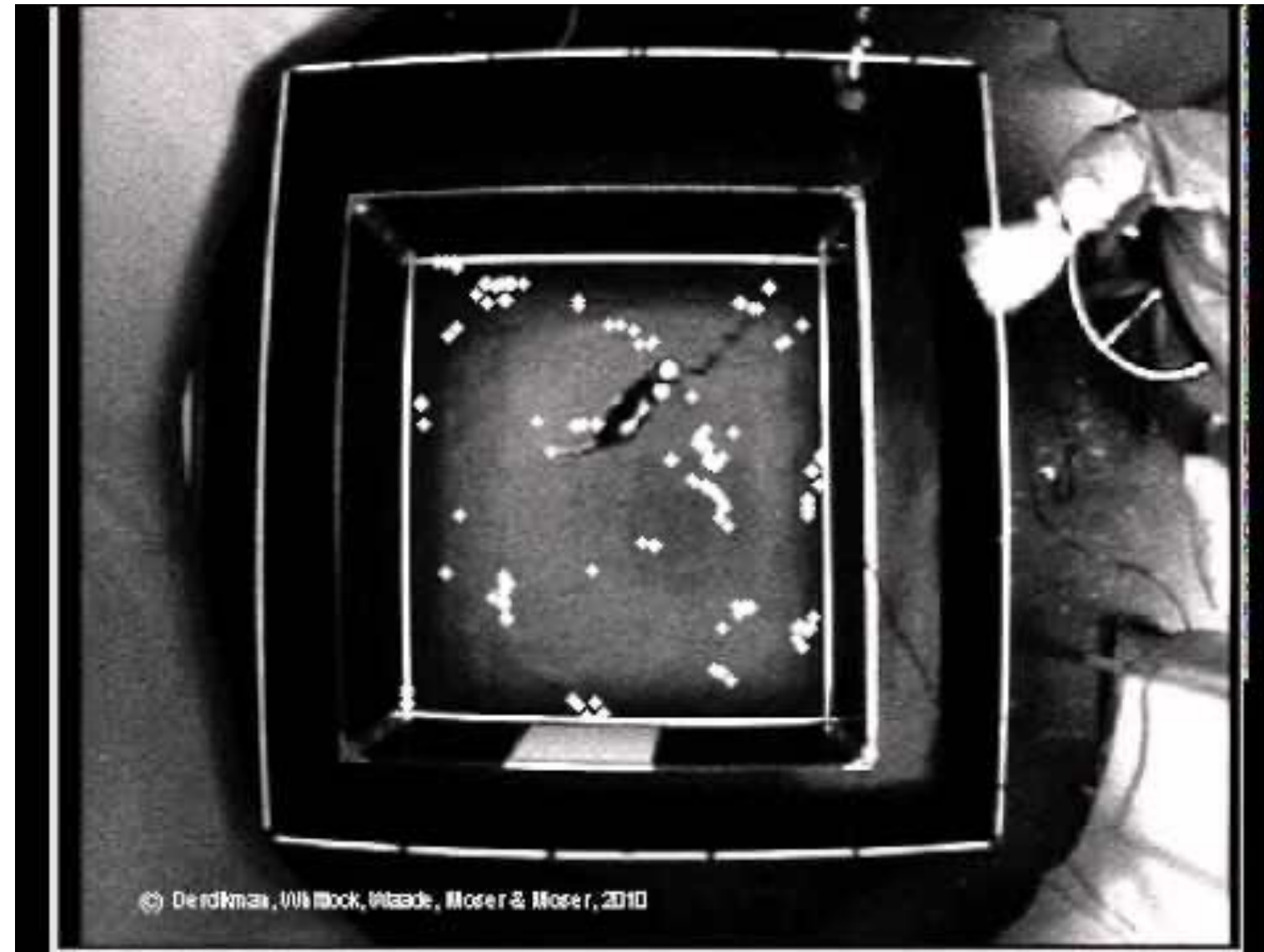
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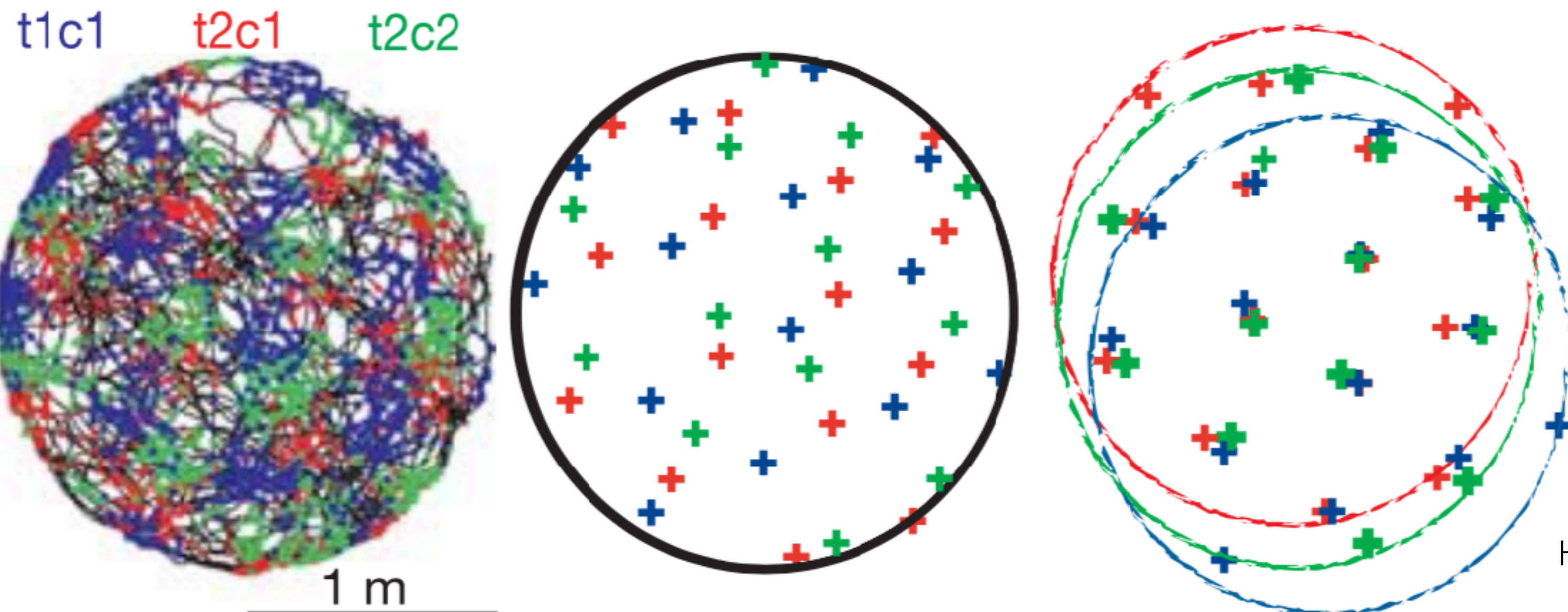
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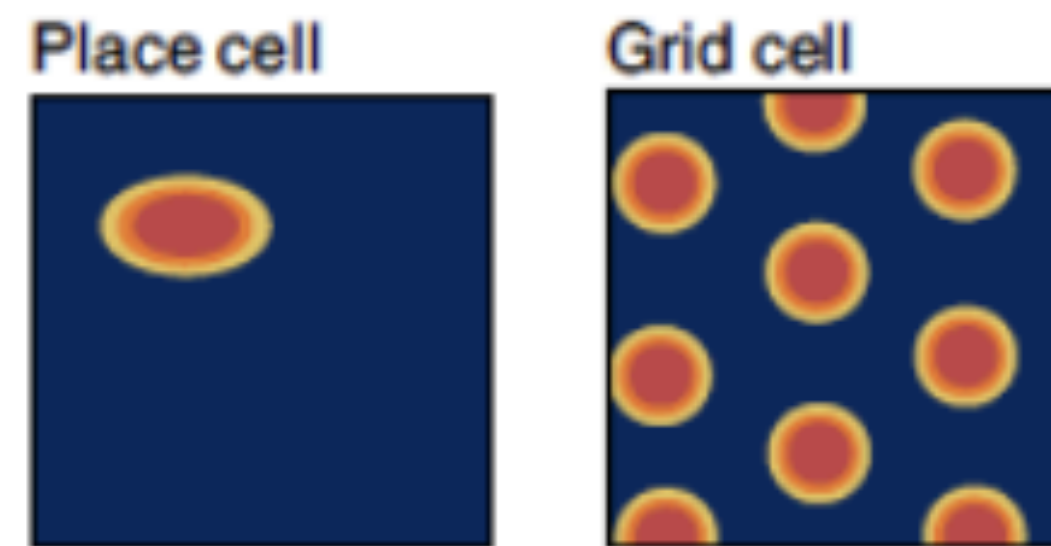


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Hafting *et al* (Nature, 2005)



# “Hippocampal Zoo”

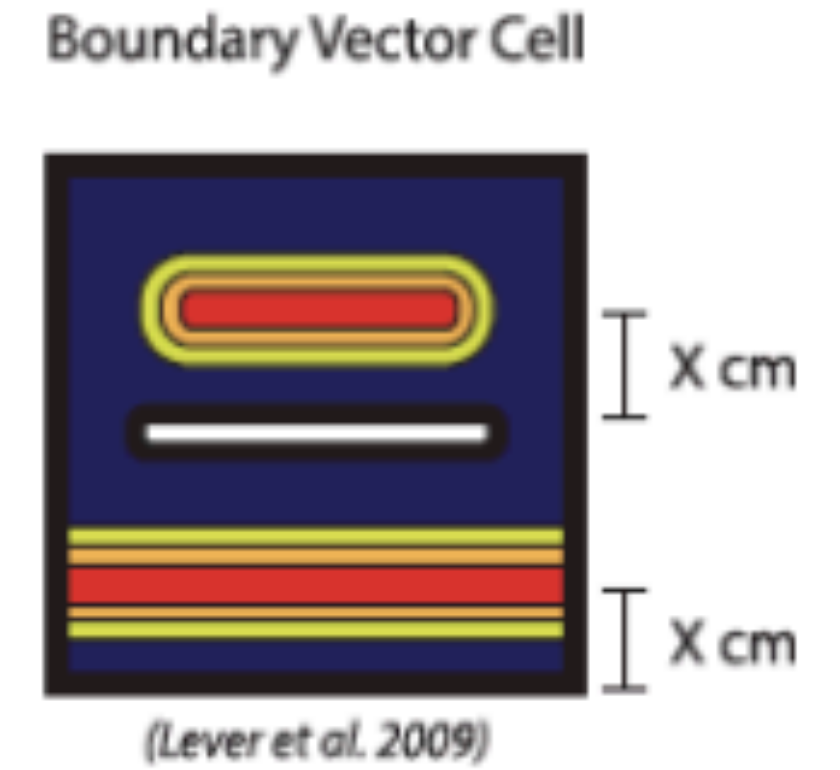
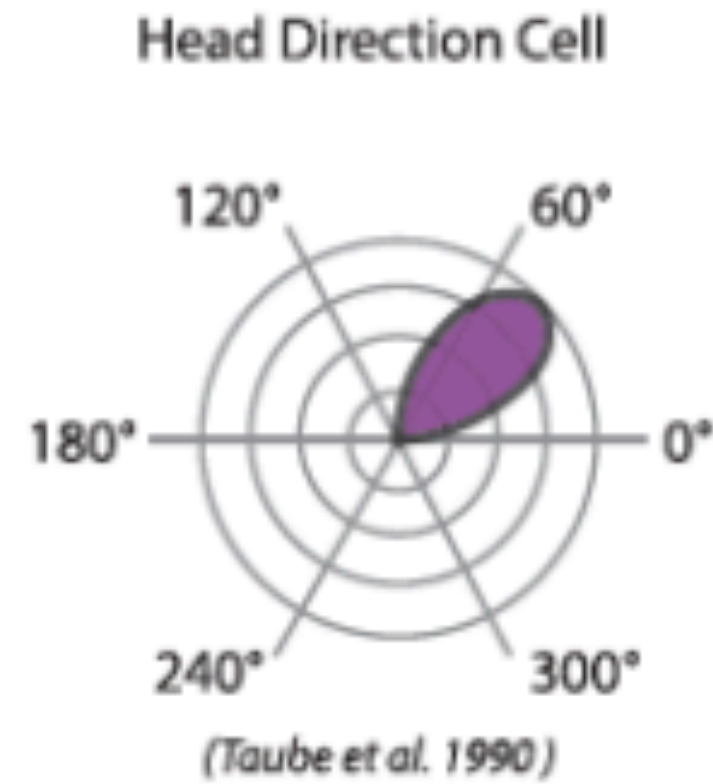


Whittington et al., (2022)

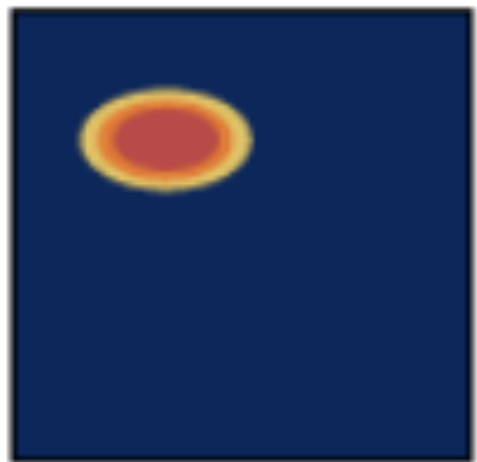
Behrens et al., (2018)



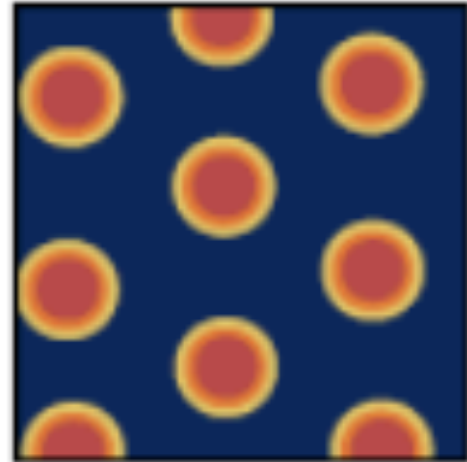
# “Hippocampal Zoo”



Place cell



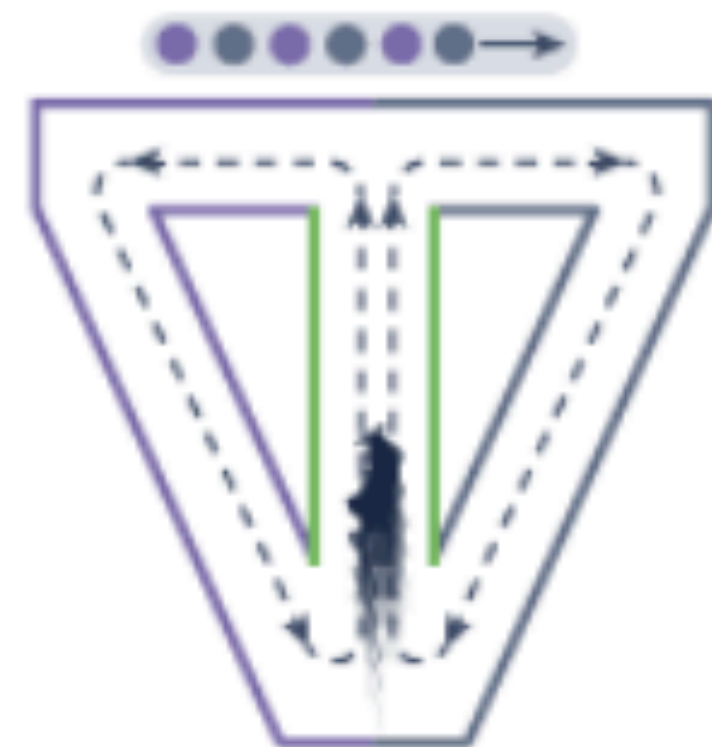
Grid cell



Border cell



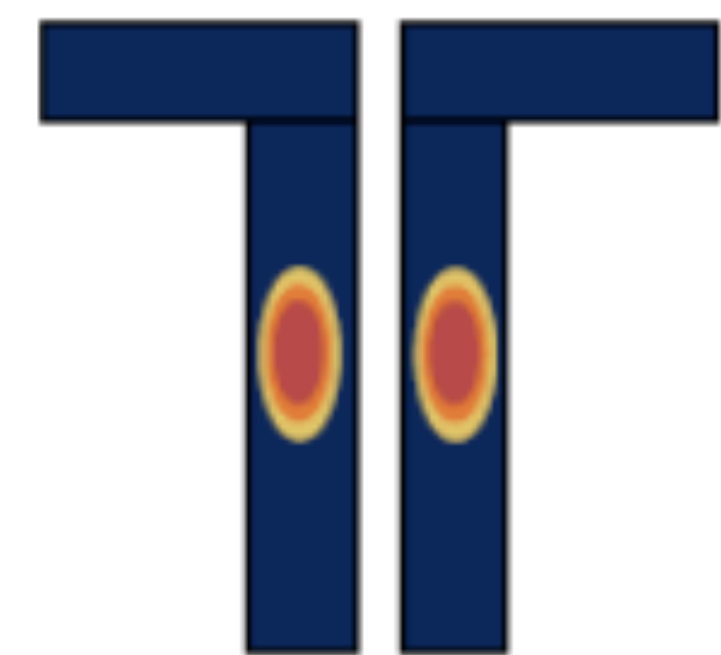
Object-vector cell



Splitter cell



Place cell



Whittington et al., (2022)

Behrens et al., (2018)

# Tools for navigation

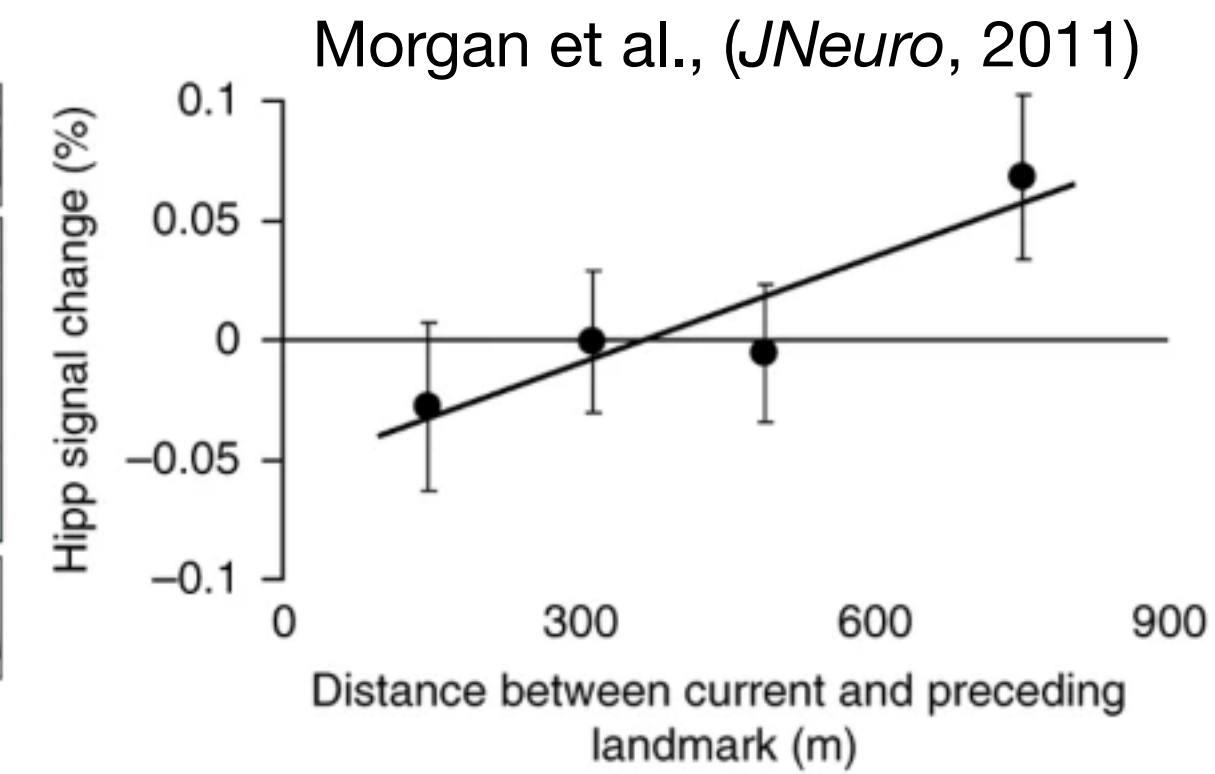
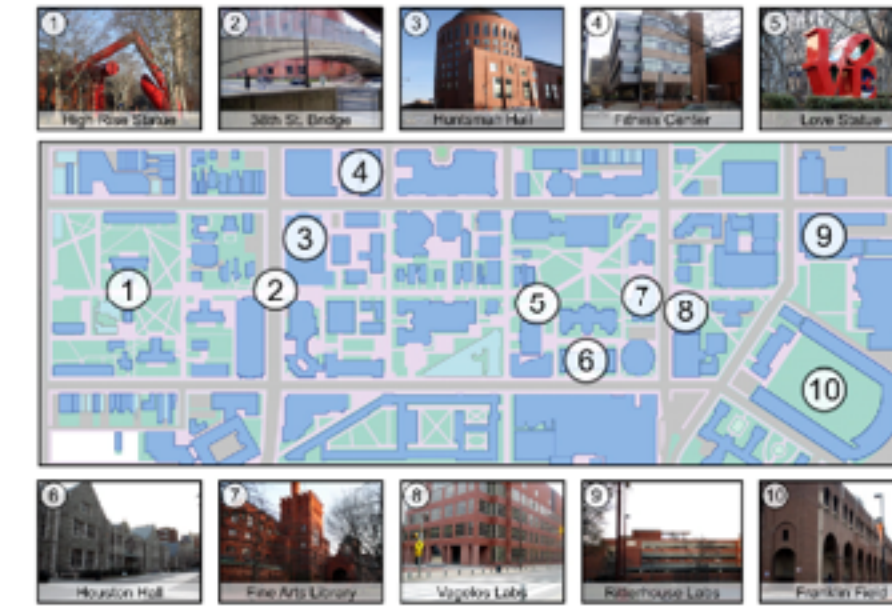


- The **Hippocampus** represents spatial distance between landmarks (Morgan et al., 2011) and between events (Nielson et al., 2015)
- The **Entorhinal Cortex (EC)** encodes the direction of travel (Doeller et al., 2015)
  - Participants moved in a VR environment
  - When direction aligned with one of the 3 axes of their grid cells, we observe stronger BOLD activation in the EC
  - These angles are remarkably robust, and are preserved (in the same environment) when participants return to the scanner days or weeks later

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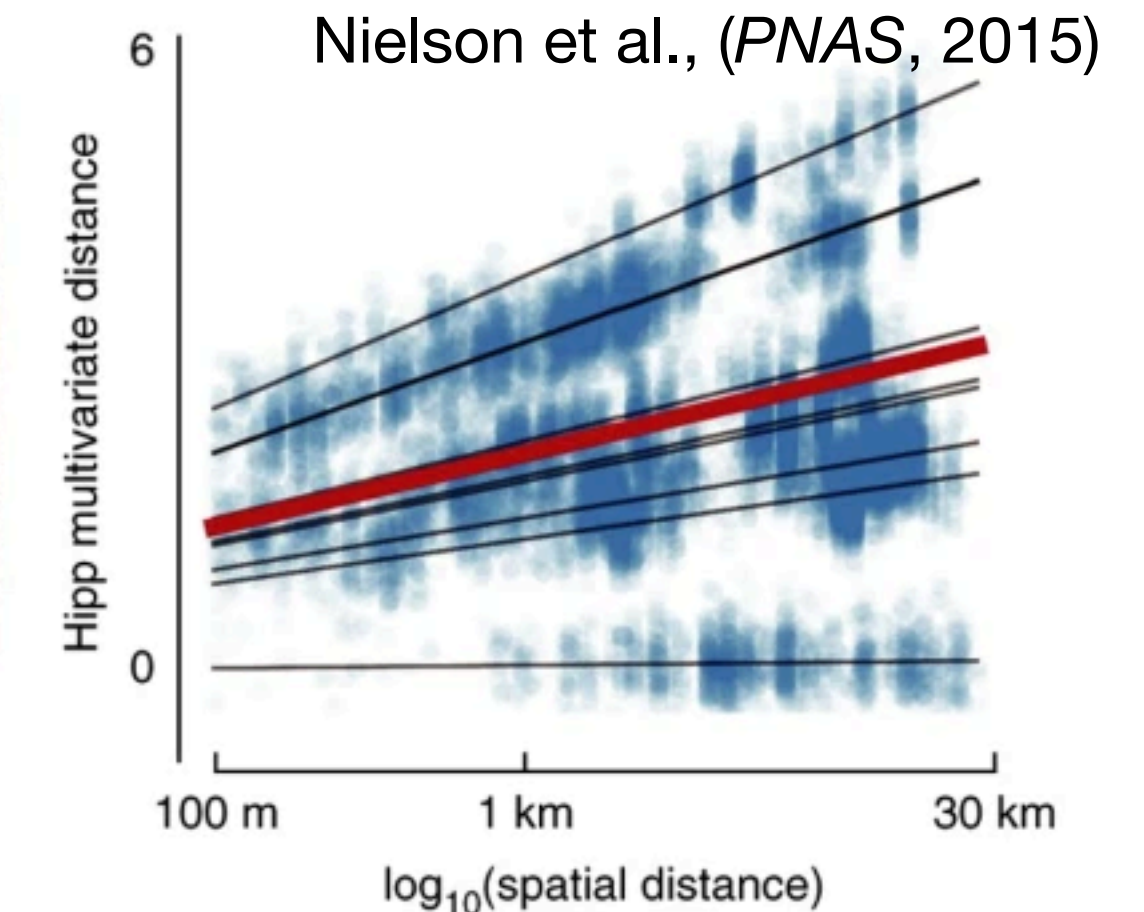
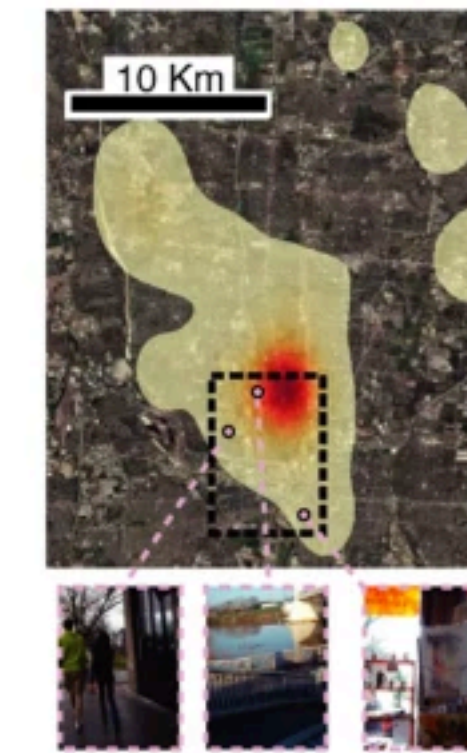
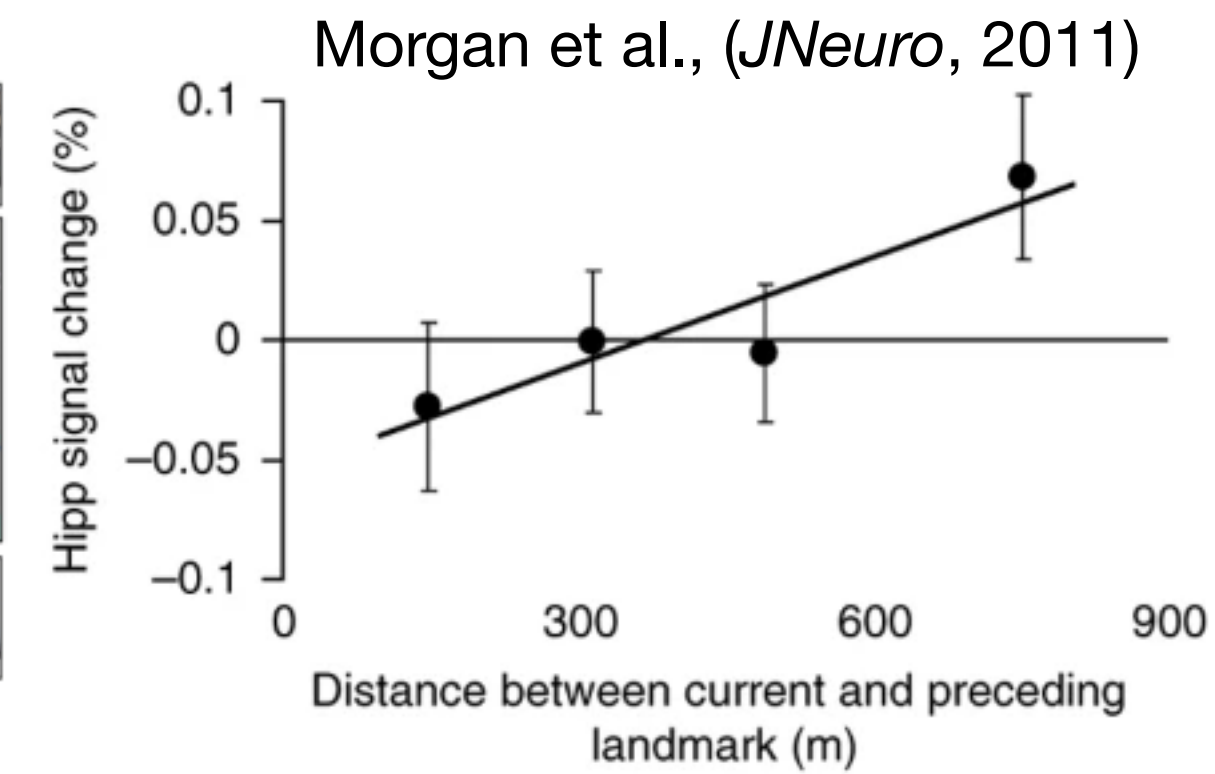




# Tools for navigation

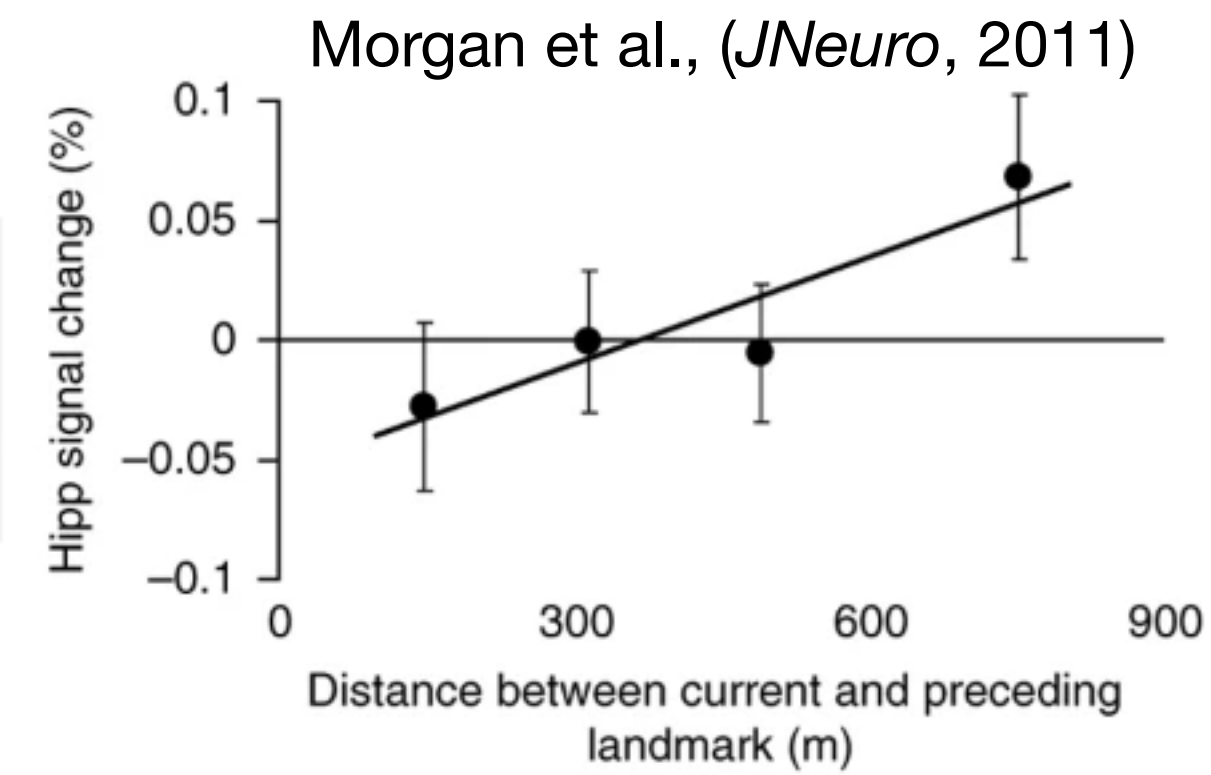


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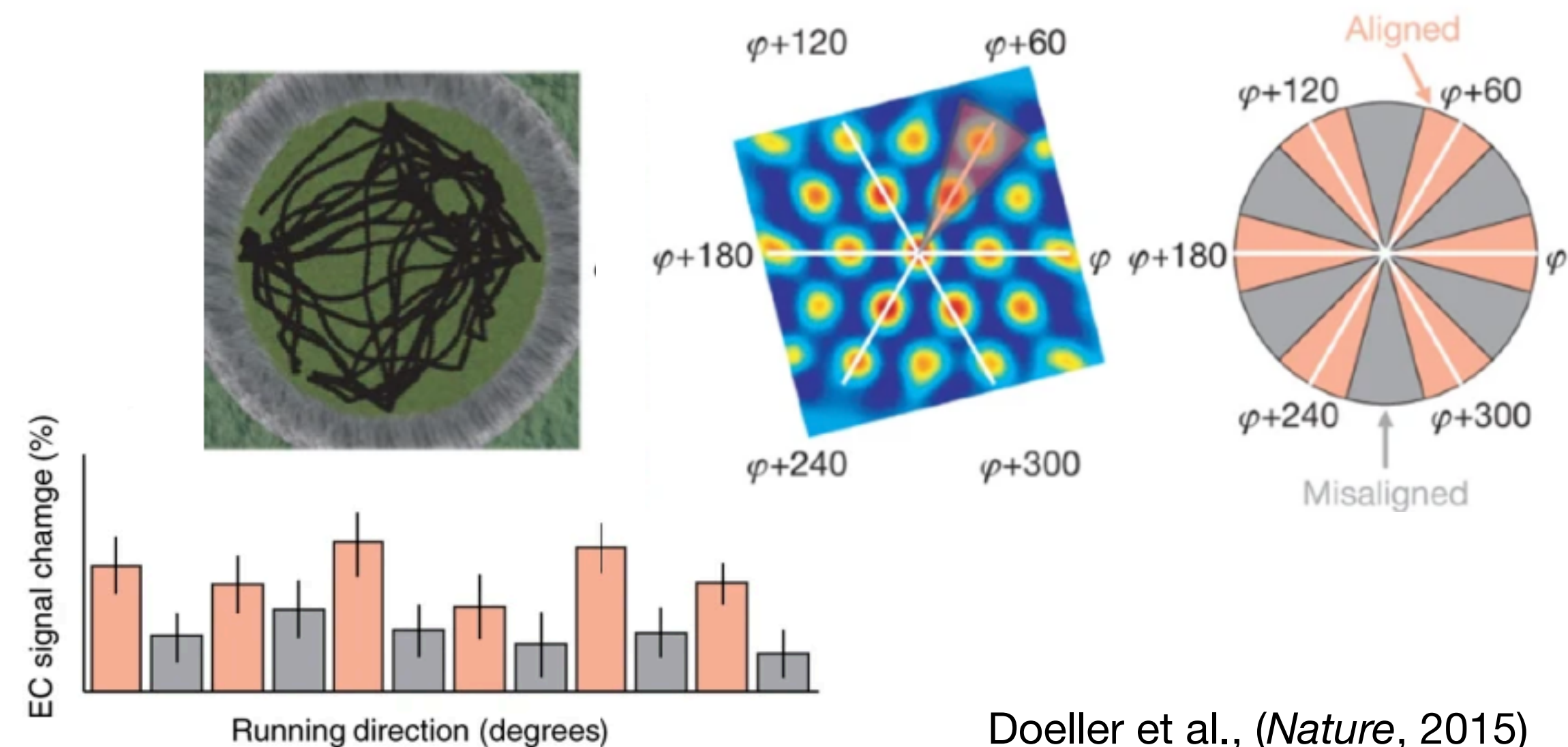
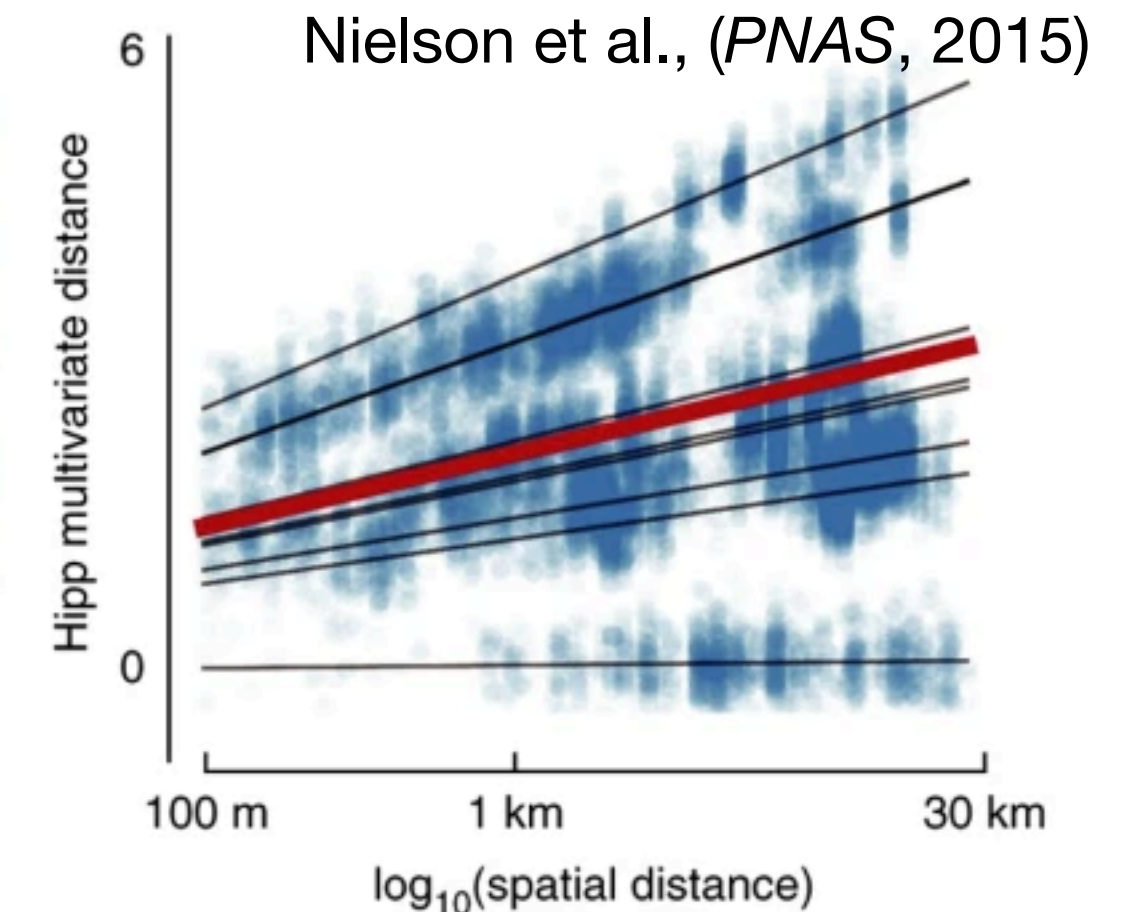
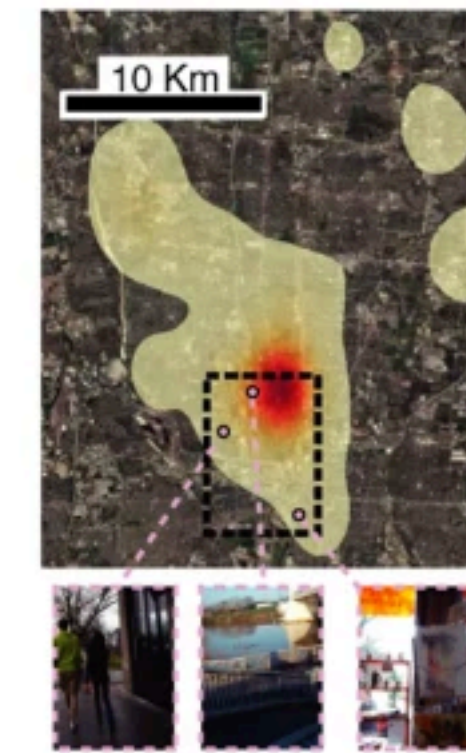
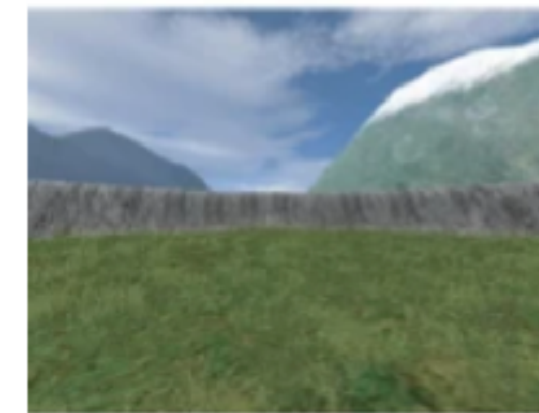
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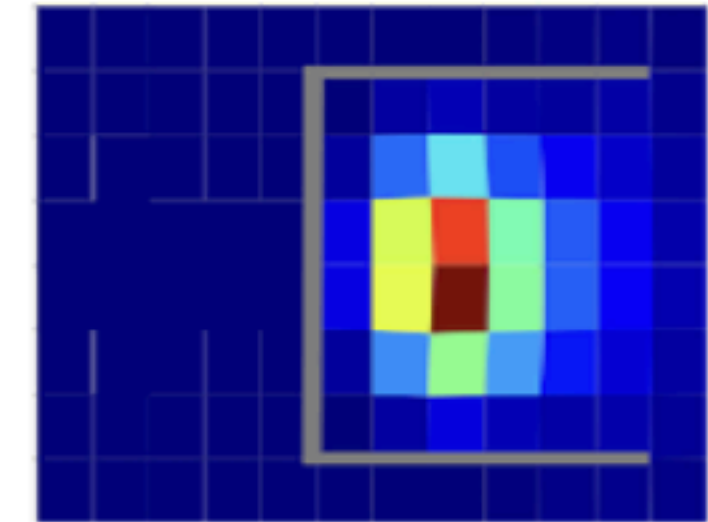
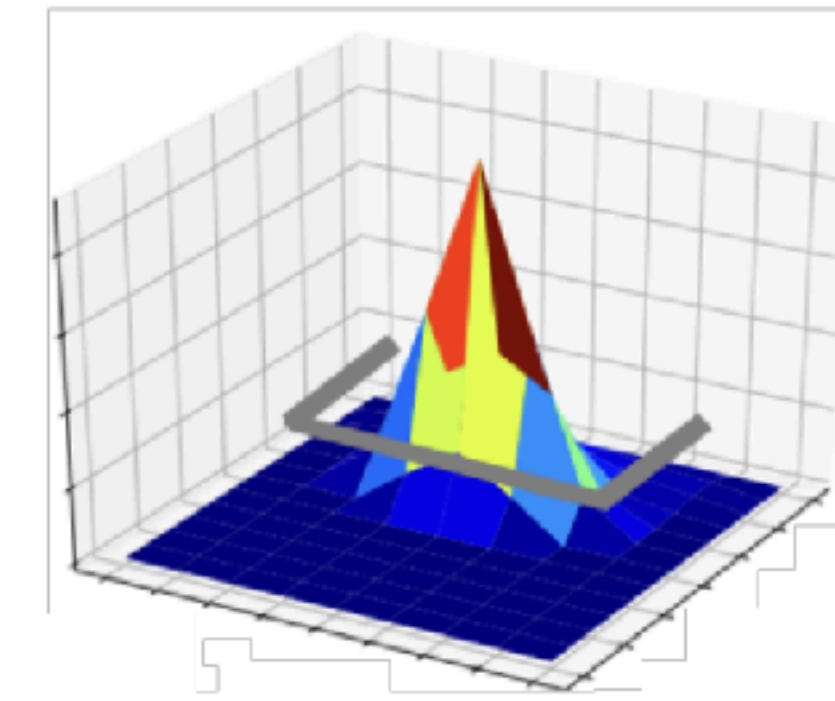
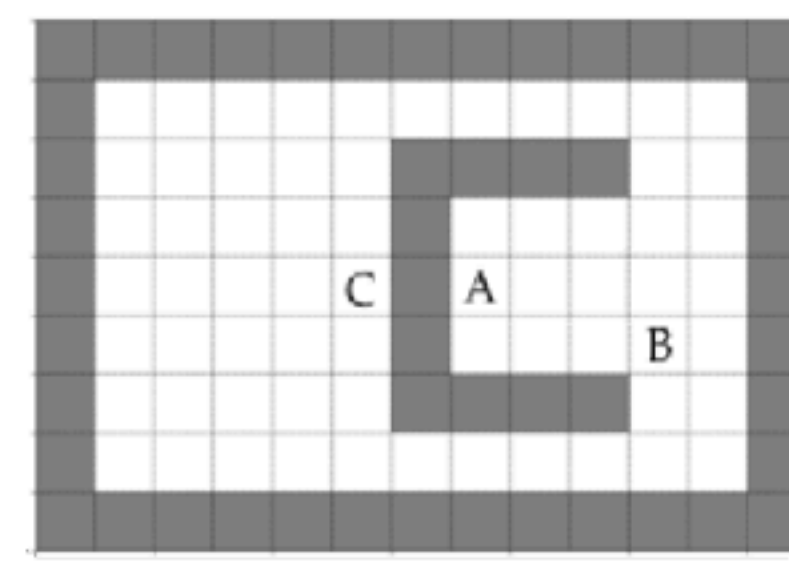




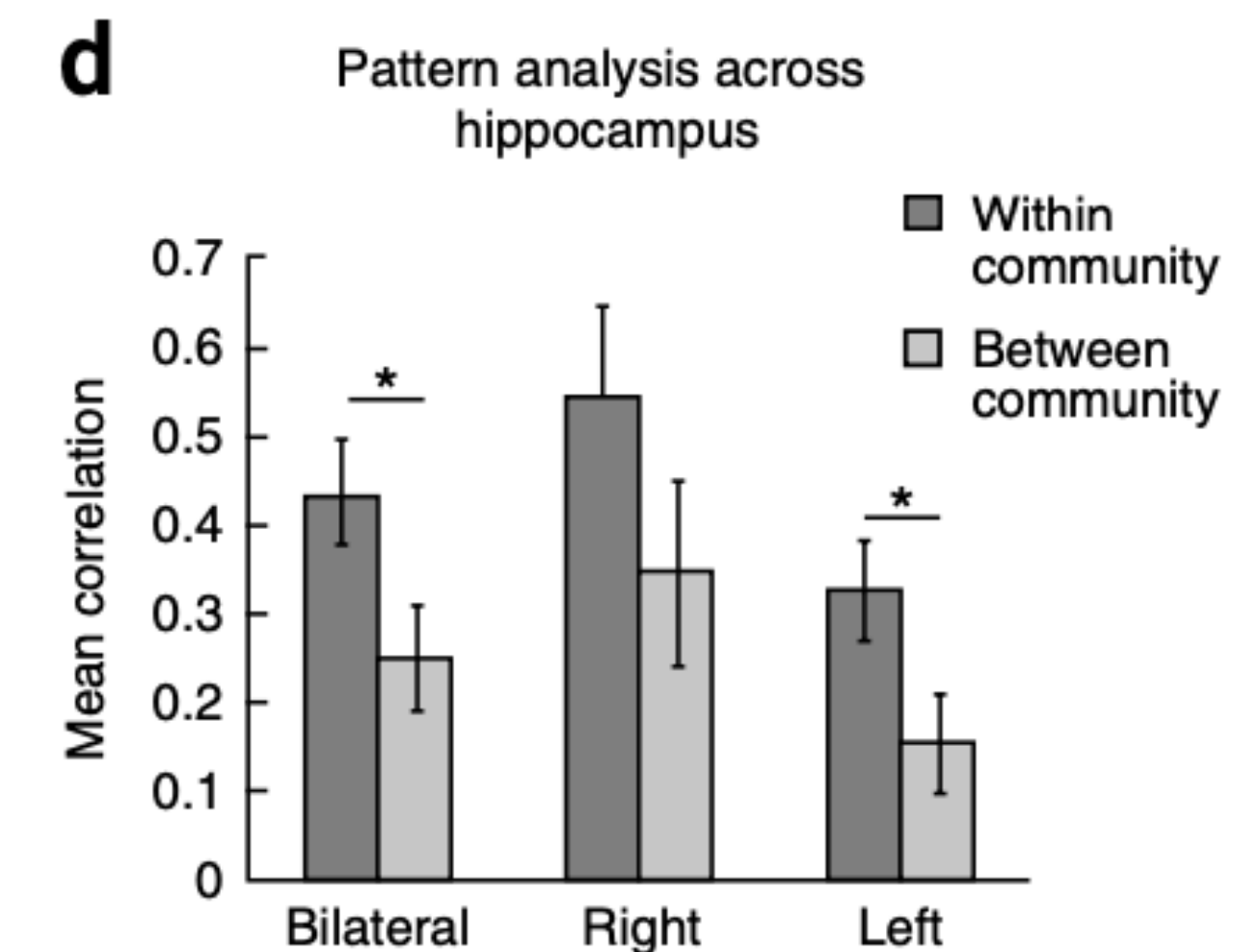
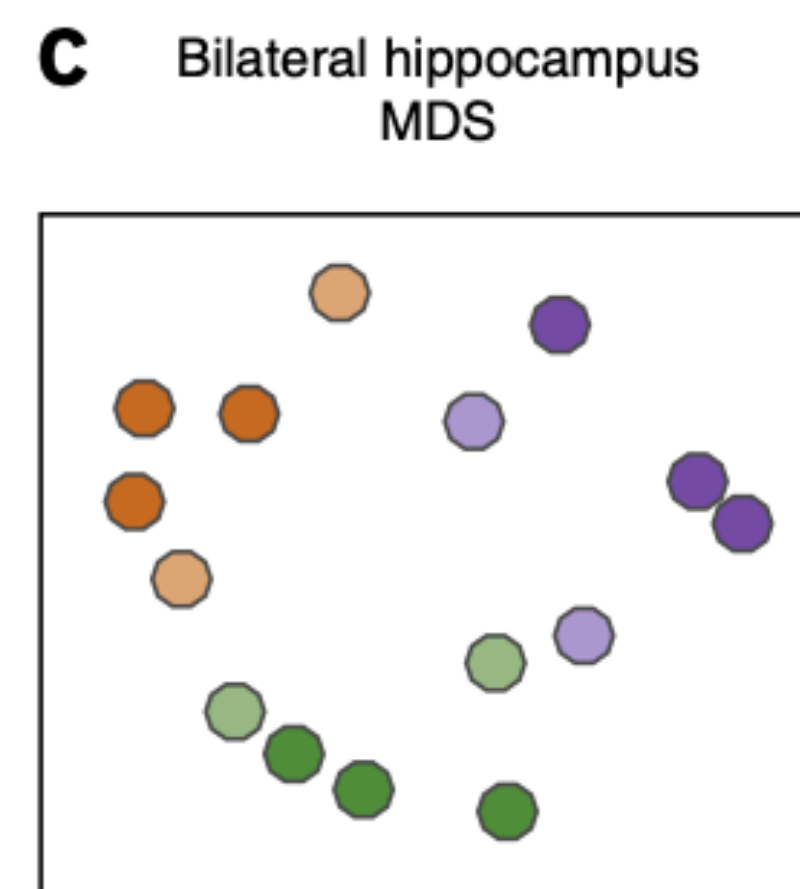
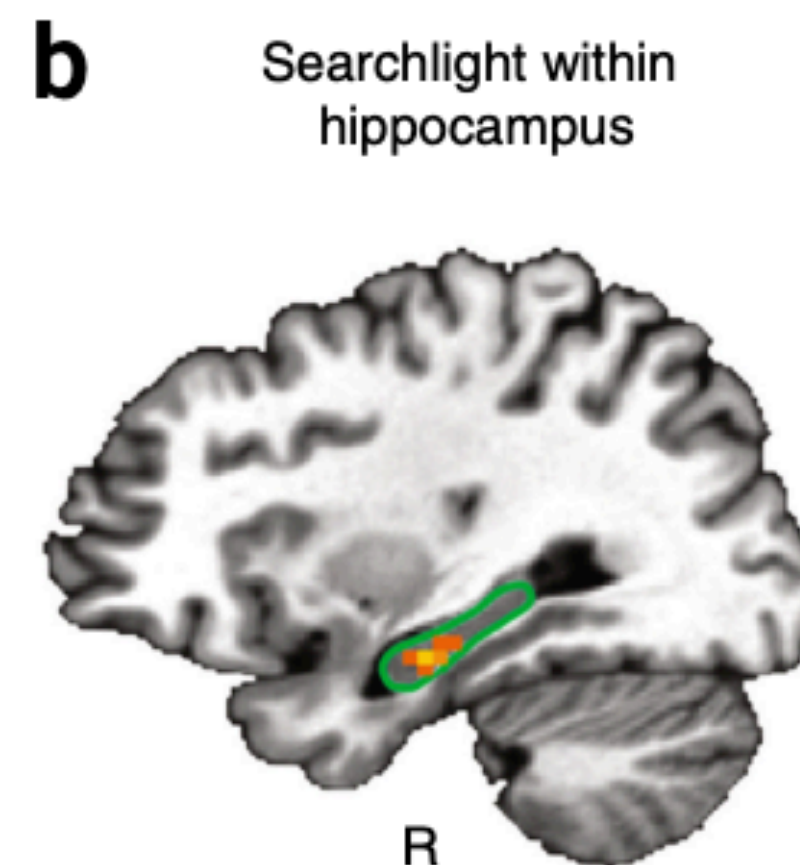
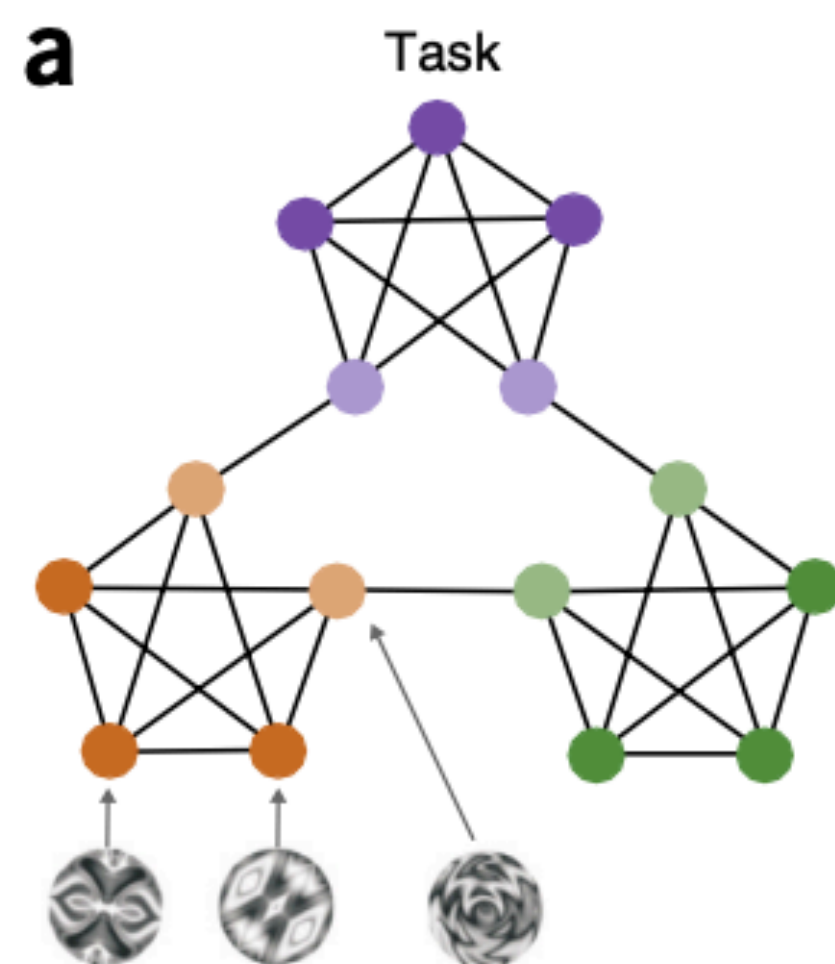
# Not just naïve distance, but based on the structure of the environment

- As in Tolman's experiments, the brain represents distance in the environment based on the transition structure
- Not just "as the crow flies" but a structure-informed distance metric

A = goal



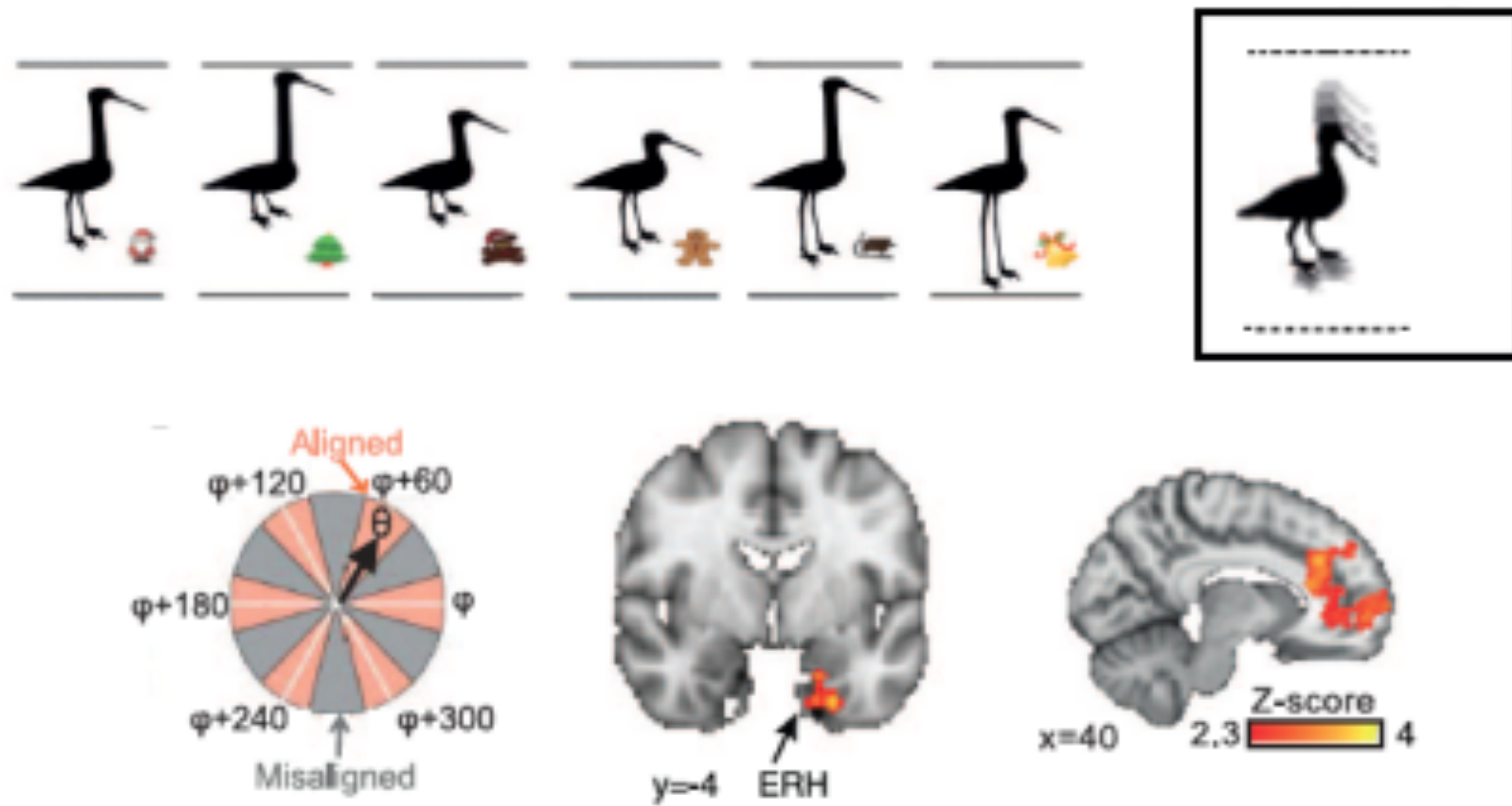
Machado et al. (ICLR 2018)





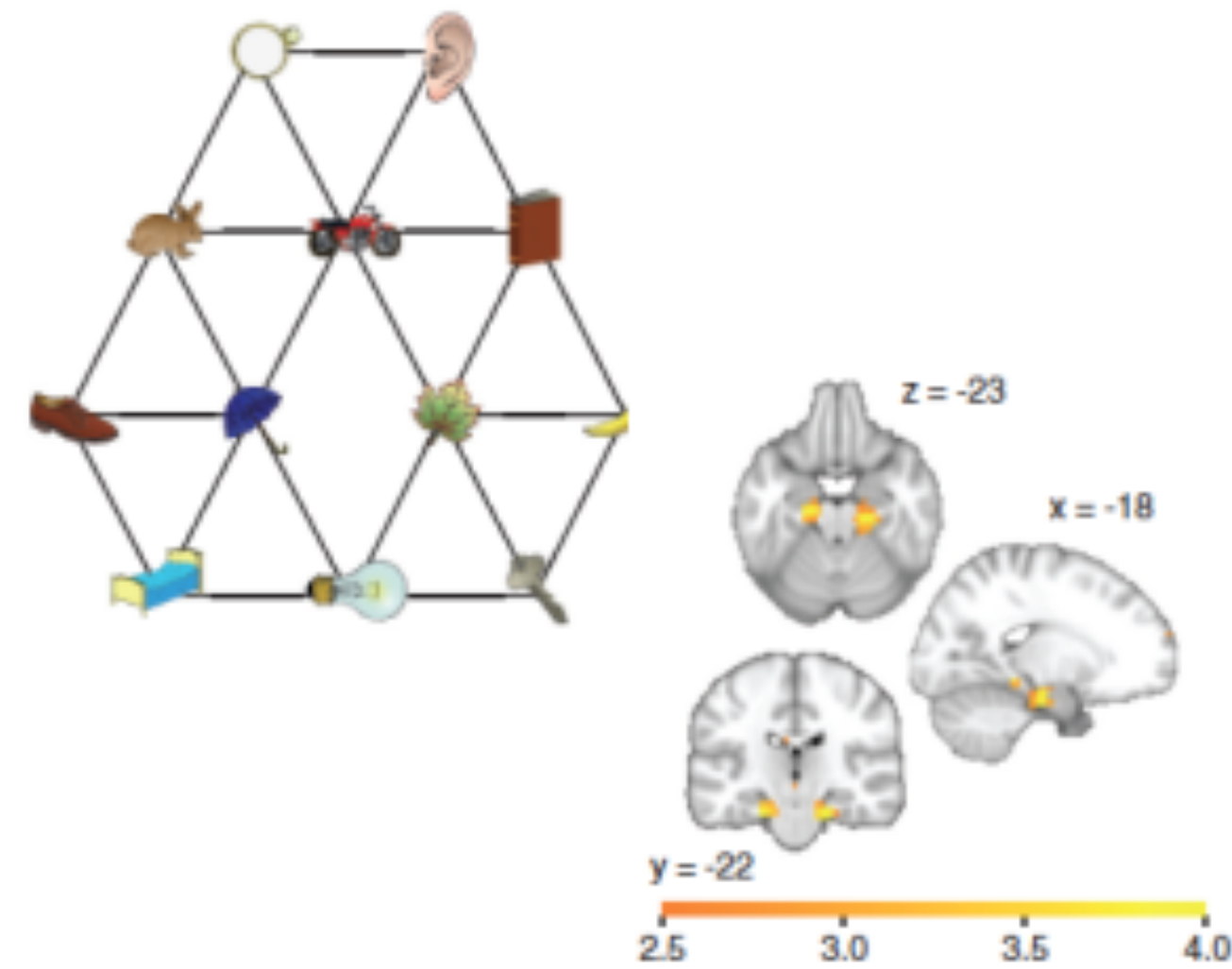
# Not just spatial, but also conceptual navigation

## Abstract features



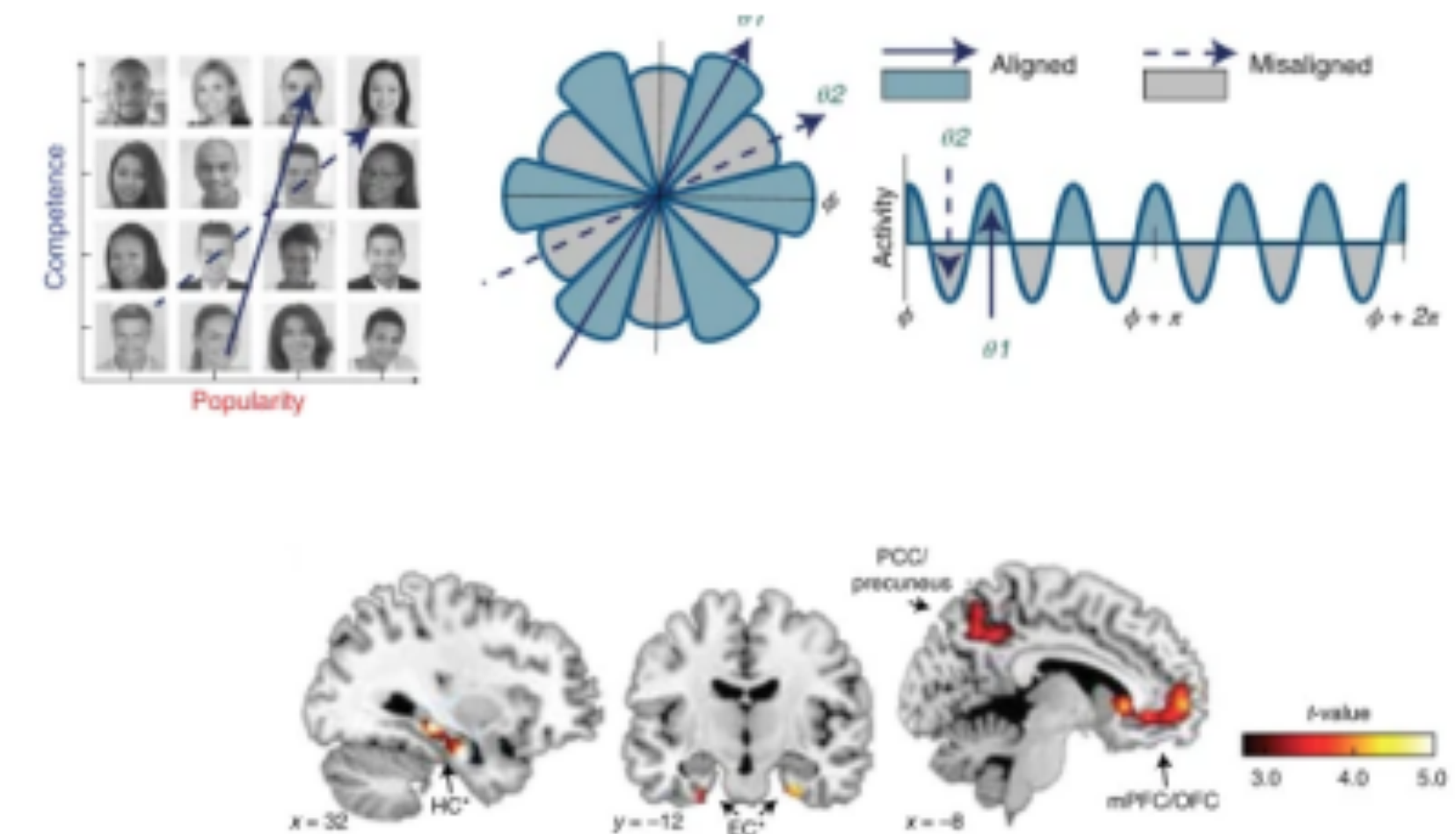
Constantinescu et al., (*Nature* 2016)

## Relational structure



Garvert et al., (*eLife* 2017)

## Social Hierarchies



Park et al., (*NatNeuro* 2021)

# Do we always need a representation of the environment?

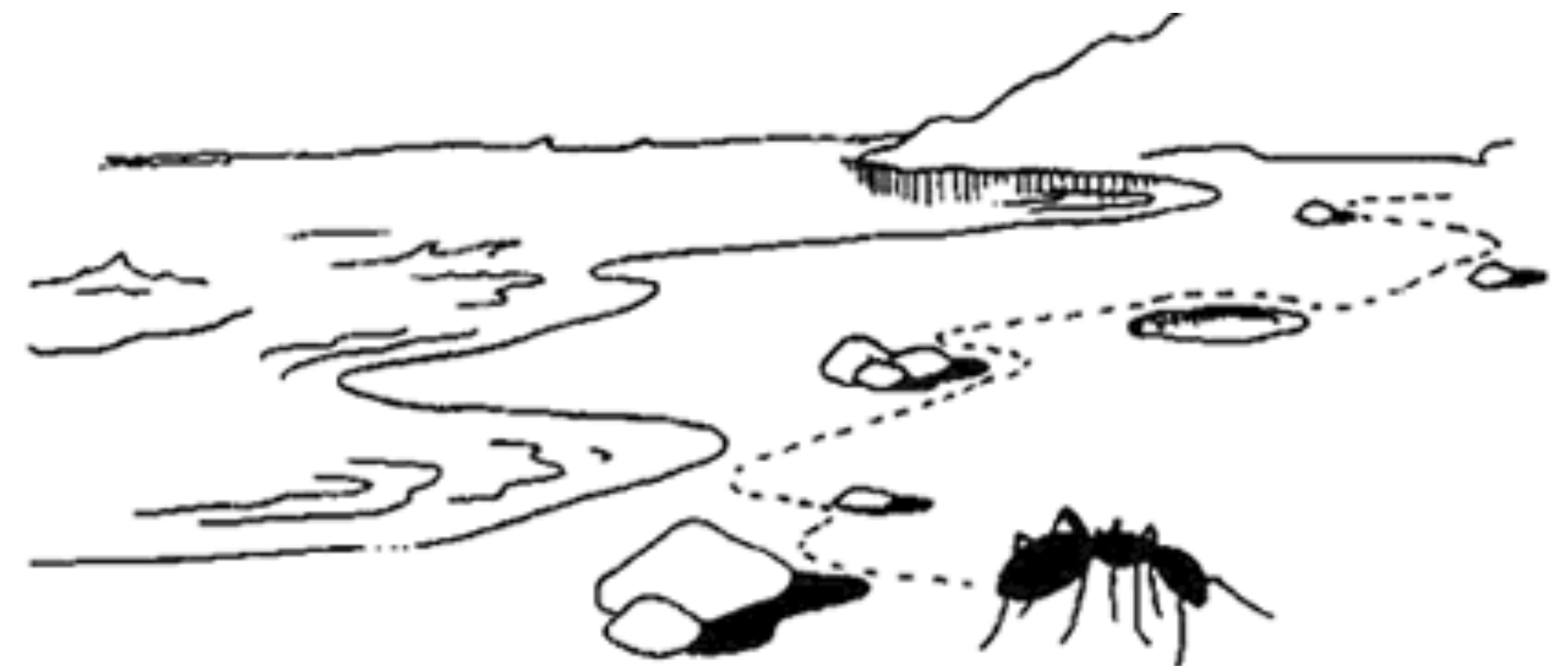
*An ant, viewed as a behaving system, is quite simple. The apparent complexity of its behavior over time is largely a reflection of the complexity of the environment in which it finds itself. I should like to explore this hypothesis with the word “man” substituted for “ant.”*

- Herbert Simon (1970)



Herbert Simon

Grandfather of AI  
and proponent of  
Bounded Rationality





# Cognitive Maps: Summary

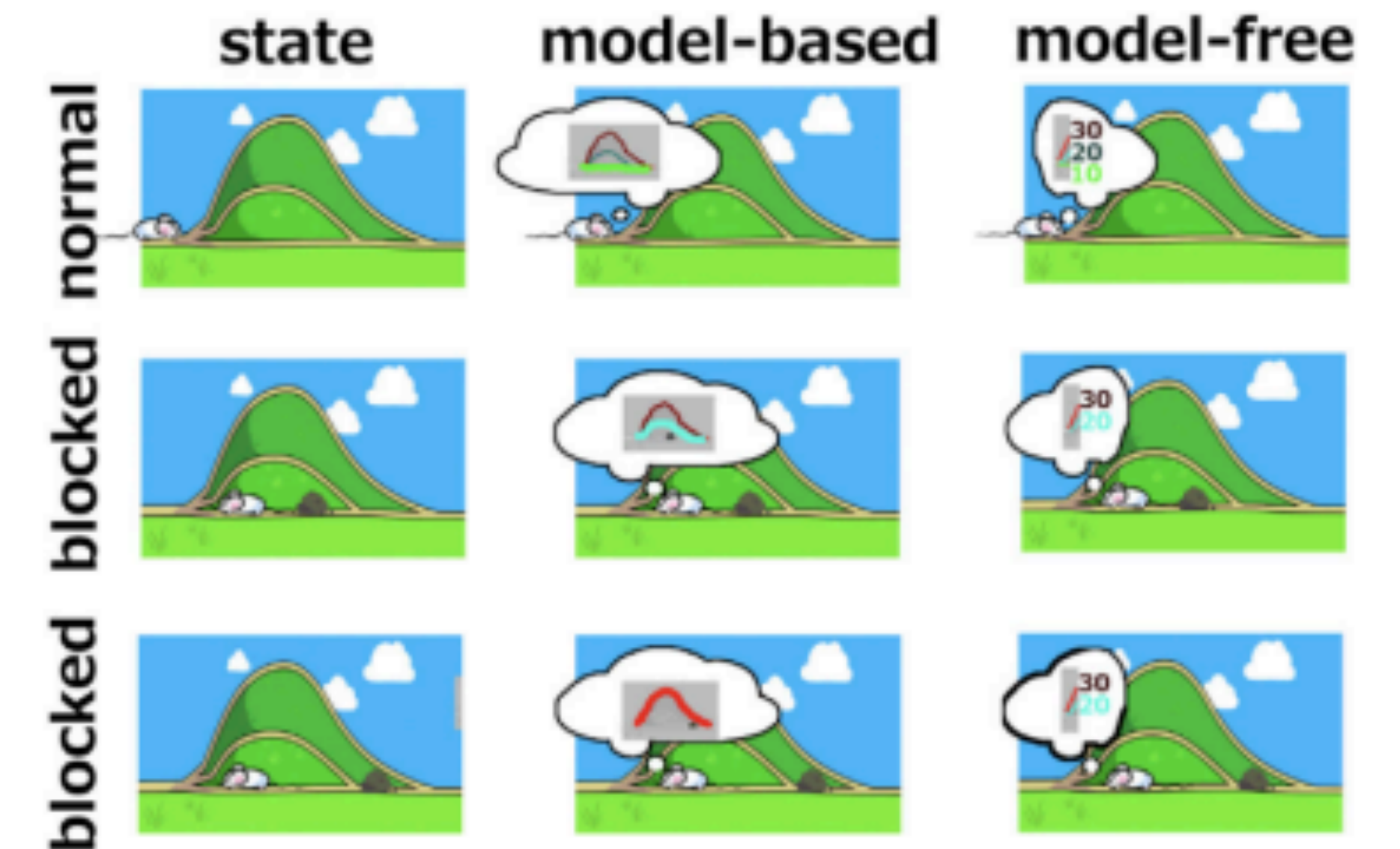
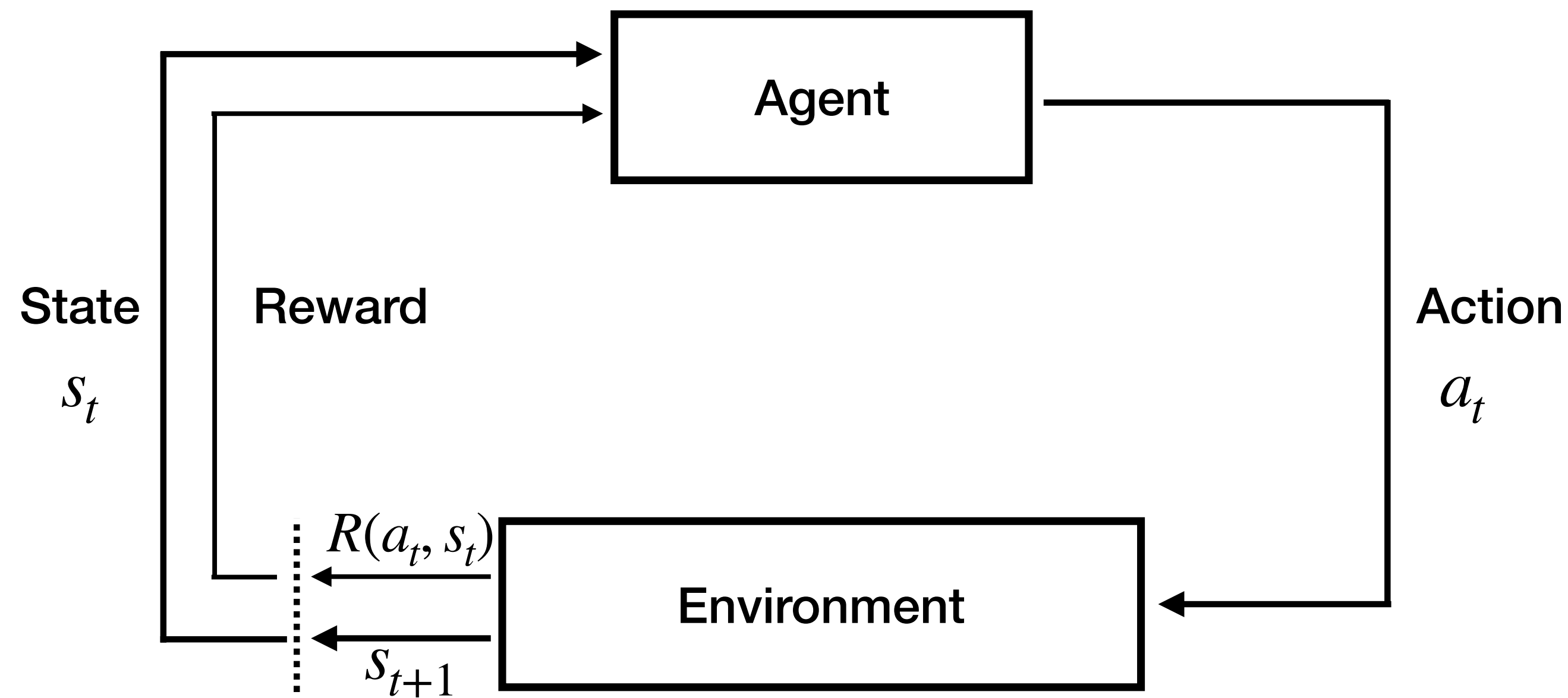
- Learning is more than just a telephone switchboard of Stimulus-Response (S-R) associations
- We learn a **map-like representation** of the environment, allowing us to rapidly generalize and plan efficiently
  - Tolman refers to this as **S-S learning**
- Neural evidence for a cognitive map in the brain
  - **Place cells** in the Hippocampus encode location and distances
  - **Grid cells** in the Entorhinal Cortex provide a coordinate system and encode direction of travel
  - + a whole zoo of other specialized cells in the hippocampal-entorhinal system
- Cognitive maps are sensitive to **transition structure** and used in abstract, **conceptual** contexts as well

# General principles

- **Symbolic AI:** Learning as inferring rules and manipulating symbols
  - In contrast to *subsymbolic AI* (i.e., neural networks), which learn by updating associating weights
  - For symbolic AI, learning corresponds to search over hypotheses, but current solutions are intractable/inefficient in most interesting settings
    - How do people manage to learn symbolic rules/programs efficiently?
- **Cognitive maps:** Learning as inferring a representation of the structure of the environment
  - Not just S-R relationships but also S-S latent learning
  - Do we always need a representation of the environment?
- Both lines of research capture mechanisms for learning **structure**
  - Structure as the relationships between different symbolic concepts
  - Structure as the relationship between stimuli in the environment
- Is there a common basis for both forms of learning? Or are they complementary systems?



# Next week: Introduction to Reinforcement Learning



**Model-based and model-free decision making** in a cartoon of a maze invented by Tolman and Honzik (1930)